VOLUME XXXIV JULY, 1924 NO. 3

THE MONIST

NUMBER: AN INTRODUCTION TO THE THEORY OF ANALYTIC FUNCTIONS

THE following memoir is intended to form the first three chapters of an exposition of Weierstrass' theory of analytic functions. The exposition, however closely it follows Weierstrass' train of thought, is, nevertheless, not a reproduction of his lectures. I have endeavored before anything to bring out clearly the connection between the point of view fundamental with Weierstrass and Cantor's later work.

The view here taken of the concept *Whole Number* is not, I should imagine, the same as that of Weierstrass. A lithographed pamphlet, *Einleitung in die Arithmetik, Erster Teil,* which contains one of the most authentic accounts of a course of lectures by Weierstrass, begins with the following words: "Arithmetic assumes nothing but the concept of Number. We make clear to ourselves what Number is by realizing what we do when we count, for Number is the result of counting." [1] In a footnote reference is made to a work of Zeller, *Logik und Erkenntnistheorie*, I. §13.

This seems always to have been Weierstrass' fundamental view. It appears to me, however, that the train of thought here put forward conforms better with the true essence of Number: for this is independent of all outer experience. Among other things, it has the advantage of exposing, as immediately clear and given *a priori*, the con-

[1] "Die Arithmetik hat keine andere Voraussetzung als den Begriff der Zahl. Was aber die Zahl ist, machen wir uns klar, indem wir uns vergegenwärtigen, was wir thun, wenn wir zählen, denn die Zahl ist das Resultat des Zählens."

cept of Infinity at the same time as that of Number. If, on the other hand, in establishing the number-concept, one sets out from the concept of *counting*, which is derived from *outer* experience, Infinity remains unexplained; by this omission Weierstrass' system loses its most important foundation.

After Cantor, the question has been much debated, how far the fundamental concept of *Whole Number* could be replaced by a more general, more comprehensive, primitive concept such as, for example, that of *Aggregate*.[2] However interesting and attractive the attempts are, which owe their existence to efforts to throw light on this question,[3] they appear to me to be mistaken from the beginning. *Aggregate* is not, like *Number*, a concept which is given *a priori*. It presupposes a definition, in the framing of which one has to appeal to other concepts which are just as little given *a priori*. The same is the case with all other attempts to replace number by any other fundamental concept of thought.

I have given expression to this view in the words: "Number is the beginning and the end of Thought; with Thought was Number born, and Thought does not reach beyond Number." [4] which I had cut in stone at the entrance to the home for mathematical research that has been built at Djursholm.[5]

Another, though unessential, deviation from Weierstrass' treatment occurs in the definition of and exposition of the laws for incommensurable numbers; for what I denote by the group-sign (( )) he uses the sign of summation, which is already employed in analysis. I have

[2] German: *Menge;* French: *ensemble.*

[3] For example, cf. E. Borel: *Leçons sur la théorie des fonctions*, 2nd ed., Note IV. Paris, 1914.

[4] Die Zahl ist Anfang und Ende des Denkens
Mit dem Gedanken wird die Zahl geboren
Ueber die Zahl hinaus reicht der Gedanke nicht.

[5] Mittag-Leffler Mathematical Institute. See *Acta Mathematica*, Vol. 40, 1916, pp. III-X.

chosen the symbol (( )) in order that the trained mathematician may the more easily avoid those false conclusions to which a system of symbols with the use of which, in another sense, he had already familiarized himself, might lead him.

CHAPTER I

The Whole Number

1. *Number—Whole Number*—is a simple concept which is given *a priori* and cannot be defined by word or image, and which, of itself, constitutes the fundamental form of thought.

The concept of whole number arises by means of inward contemplation in the following way: we present to the imagination a certain thing, the *unit,* and while retaining this, we add to it again the same thing, whereby the *second* number arises, and then once more the same thing, whereby the *third* number comes into being, and so on in the same manner.

These whole numbers which are formed by means of the same unit are said to belong to the same number-system. They appear in a definite sequence, so that, for every number, we can say which is its *predecessor* and which is its *successor.* To every number, except the first, there corresponds another number which immediately precedes it; and to every number, including the first, there corresponds another number which immediately succeeds it.

The process by means of which numbers are formed has no end; we can always continue it beyond the stage at which we may have stopped—no number exists which has not a successor. In this fact, we are met at once, at the very beginning of thought, by the *concept of infinity,* and indeed by this concept in its original form—namely as the

mathematical concept of infinity—the only concept of infinity that can be grasped by thought. This concept of infinity is given with and by means of the concept of whole number, and appears with the same clearness as the concept of number itself.

We establish in the following manner the concepts of *equality* and *inequality* concerning numbers which belong to the same number-system.

Two numbers a and b are said to be *equal* to one another, if and only if they are the same number. We denote this by $a = b$ or $b = a$. On the other hand, the numbers are said to be *unequal* to one another when they are not the same number. In symbols $a \neq b$ or $b \neq a$. If a precedes b, or b succeeds a, we denote this by $a < b$ or $b > a$.

2. We have so far supposed the unit to be a certain quite arbitrary thing. If we give this thing no other attribute than that of being embodied in the mind in the formation of number, we denote the unit by *One* (1), and we call those numbers whose unit is One *absolute numbers.*

The concepts *greater* and *less* are not given *a priori* for absolute numbers and derived from some general theory of magnitude, but, on the contrary, they are obtained by means of definition; and, indeed, in such a way that, of two numbers which are unequal to one another, the preceding one is the *less* and the succeeding one is the *greater.*

The theorem, "no number exists which has not a successor," can therefore be worded for absolute numbers as follows: "no number exists which is greater than all the others."

3. When once we are in possession of the concept of whole number we easily arrive, by definition, at the concept of the *sum of two numbers.* "The sum of two numbers is defined to be that number which we obtain when we continue the process of repetition of the unit already carried through to form the first number just so many times

as the number of repetitions of the unit that is necessary to obtain the second number." We can also express this as follows: "The sum of the given number and the unit is the number immediately following the given number. The sum of two numbers, of which the latter is not the unit, is the number immediately following the sum of the first number and of the number immediately preceding the second." It follows immediately from our definition of sum that: "The sum of two numbers is always the same unambiguously given number."

The definition of sum appears formally as a single definition, but it is in reality an unlimited sequence of definitions, namely, one definition for each combination of numbers. This is a state of things that will constantly occur in what follows and which, among other things, provides mathematical analysis with the means of continually drawing into its domain new regions of unlimited extent.

4. Now that we have established the concept of sum, we are in a position to deduce two main theorems, from which the remaining properties of a sum immediately result. The first main theorem, which contains the so-called *Associative Law,* reads:

"If a, b, c are three given numbers, the sum of the number a and the number $b + c$ (which latter is the sum of b and c) is equal to the sum of $a + b$ (i. e., that number which is the sum of a and b) and the number c; or in symbols:

$$a + (b + c) = (a + b) + c$$

For let e be the unit of the numbers in question. If the number c represents the unit e, the Associative Law contains nothing other than a repetition of the definition of the sum of two numbers. Let us now suppose that the law holds for the numbers a, b and c. Then it holds also for the numbers a, b and $c + e$, i. e., we have

$$a + [b + (c + e)] = (a + b) + (c + e)$$

For

$$\begin{aligned} a + [b + (c + e)] &= a + [(b + c) + e] \\ &\qquad \text{(Def. of sum)} \\ &= [a + (b + c)] + e \\ &\qquad \text{(Def. of sum)} \\ &= [(a + b) + c] + e \\ &\qquad \text{(Our hypothesis)} \\ &= (a + b) + (c + e) \\ &\qquad \text{(Def. of sum)} \end{aligned}$$

But the law holds when c is the unit e; consequently it holds also when c represents the number $e + e$, and therefore likewise when c is equal to $e + e + e$, and consequently it holds for every number c, however the numbers a and b may be chosen. We meet here for the first time a manner of deduction which is usually called "complete induction," or "deduction from n to $n + 1$." The course of this manner or method of proof is as follows: we prove first that if the theorem holds for an entirely arbitrary number n, it holds also for the immediately following number $n + 1$. But the theorem does hold for a certain number n—in our case this was two—and consequently holds for the immediately following number $n + 1$, and holds further for the next immediately following number, and so on, or, in other words, it holds for all numbers which follow the given number n.

This deduction from n to $n + 1$ is one of the most important and indispensable methods of proof in mathematics.

Our second main theorem contains the so-called *Commutative Law*, and reads:

If a and b are two numbers of the same number-system, the sum of a and b is equal to the sum of b and a; or, in symbols:

$$a + b = b + a$$

If e denotes, as before, the unit of the number-system, the theorem holds when simultaneously $a = e$ and $b = e$.

Further, it is clear that if it holds for $b = e$ and a definite a, then it also holds for $b = e$ and the number $a + e$.

For we have

$$\begin{aligned}(a + e) + e &= (e + a) + e && \text{(By our hypoth.)}\\ &= e + (a + e) && \text{(Def. of sum)}\end{aligned}$$

Since now the law holds for $b = e$ and $a = e$, it therefore holds also for $b = e$ and any arbitrary number a.

If now the law is true for an arbitrary a and a definite number b, it is true also for the same a and $b + e$. For

$$\begin{aligned}a + (b + e) &= (a + b) + e && \text{(Def. of sum)}\\ &= (b + a) + e && \text{(By our hypoth.)}\\ &= b + (a + e) && \text{(Def. of sum)}\\ &= b + (e + a) && \text{(Just proved)}\\ &= (b + e) + a && \text{(Asso. Law)}\end{aligned}$$

But the theorem holds for an arbitrary a and for $b = e$, it therefore holds also for an arbitrary a and an arbitrary b, as was to be proved.

5. By combining the Associative and Commutative Laws we obtain without difficulty:

"If certain given numbers are to be summed, it does not matter in what order the summation is performed. The final result is always the same number."

We also obtain:

"If certain given numbers are to be summed, we may divide the numbers into groups in any manner we please, form the sum of the numbers entering into each group (in case the group contains more numbers than one), and then sum all the numbers so obtained. The final result is again always the same number."

6. After we have treated the concept of sum, it is easy to establish the concept of the *difference* of two numbers. For we easily obtain the theorem:

"If two unequal numbers are given, there always exists, and is uniquely determined a third number c, belonging to the same number-system, which is the difference between the succeeding and the preceding of the two given numbers, and which is such that the sum of the preceding one of the given numbers and this third number is equal to the succeeding one of the two numbers"; or, expressed algebraically:

"The equation $a + x = b$, in which a and b are the two given numbers and a precedes b, always possesses as solution a uniquely determined number x (= the difference of b and a)."

We note the following theorems, which follow immediately from the above.

"If a, b, c are three given numbers of the same number-system, and $a + b = c$, then $a < c$ and $b < c$."

"If a, b, c, d are numbers of the same number-system, and $a = b$, $c = d$, then $a + c = b + d$."

"If $a > b$ and $c > d$, then $a + c > b + d$."

"If $a = b$ and $c > d$, then $a + c > b + d$."

And, conversely:

"If $a = b$ and $a + c > b + d$, then $c > d$."

"If $a = b$ and $a + c = b + d$, then $c = d$."

"If $a > b$ and $a + c = b + d$, then $c < d$."

7. We now pass on to explain the concept of the *product of numbers*. We define the product of a number and an absolute number in the following manner: The *product* of a number a and an absolute number n is to be, when n is greater than One, the sum of n numbers a, and, when n is equal to One, the number a itself. We denote this product by $a.n$."

Thus if e is the unit of the number-system and n the number of repetitions of the unit necessary to form the

number, the number itself may be denoted by $e.n$. If the unit is One, the number therefore becomes $1.n$; and, since it is also denoted by n, we have the equation

$$1.n = n$$

We can now understand without further explanation the meaning of the expression

$$((a.m_1).m_2).m_3).\ldots\ldots).m_n,$$

where $m_1, m_2, \ldots, m_n$ denote absolute numbers. The number so represented may be written more simply

$$a.m_1.m_2.m_3.\ldots\ldots.m_n$$

We understand likewise the meaning of the expression

$$a.m_1.m_2.m_3.\ldots\ldots.m_n.(1_1.1_2.\ldots\ldots.1_p),$$

where $1_1, 1_2, \ldots\ldots 1_p$ are, like $m_1, m_2, \ldots. m_n$ above, absolute numbers.

Further, we see that if, $a_1, a_2, \ldots, a_n$ are numbers of the same number-system and $m_1, m_2, \ldots\ldots, m_p$ are absolute numbers, we have

$$\begin{aligned}(a_1+a_2+\ldots+a_n).(m_1+m_2+\ldots.+m_p) &= a_1.m_1+a_2.m_1+\ldots\ldots\ldots\ldots+a_n.m_1\\ &+a_1.m_2+a_2.m_2+\ldots\ldots\ldots\ldots+a_n.m_2\\ &\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\\ &+a_1.m_p+a_2.m_p+\ldots\ldots\ldots\ldots+a_n.m_p\end{aligned}$$

8. We see, further, that the Associative and Commutative Laws hold also for the products of numbers. In this case they must read:

"Let a be a given number and m and n two given absolute numbers. The product of the number $a.m$ and the number n is equal to the product of the number a and the product of the numbers m and n; or, in symbols,

$$(a.m).n = a.(m.n)."$$

This is the Associative Law, and from it follows

$$"a.m_1.m_2. \ldots . m_\rho = a.(m_1.m_2. \ldots . m_\rho),$$

where $m_1, m_2, \ldots, m_\rho$ represent absolute numbers."

The Commutative Law correspondingly takes the form: "Let m and n denote absolute numbers The product of m and n is equal to the product of n and m; or, in symbols,

$$m.n = n.m."$$

From the Commutative Law there follows the more general theorem: "If a given number of definitely given absolute numbers are to be multiplied together, it does not matter in what order the multiplications are performed; the final product is always the same number. Also the same number is again obtained as final product, if we divide the numbers into any groups we please, form the product of the numbers belonging to each group which contains more than one number, and then multiply these products by one another and by those numbers, if any such exist, each of which by itself forms a group."

9. We note also the following theorems, corresponding to those which, in §6, we collected together for sums of numbers. They result from what precedes without any special difficulty.

"Let m, n, 1 denote three absolute numbers, and let $m.n = 1$. If each of the three numbers is greater than One, then $m < 1$ and $n < 1$. If one of the two numbers m and n is equal to One, then the other is equal to 1; and if 1 is equal to One, then m and n are both equal to One."

We now understand by a and b two numbers of the same number-system, and by m and n two absolute numbers. We have then the theorems:

"If $a = b$ and $m = n$, then $a.m = b.n$."
"If $a > b$ and $m > n$, then $a.m > b.n$."
"If $a = b$ and $m > n$, then $a.m > b.n$."

And, conversely, the theorems:

"If $a = b$ and $a.m > b.n$, then $m > n$."
"If $a = b$ and $a.m = b.n$, then $m = n$."
"If $a > b$ and $a.m = b.n$, then $m < n$."

10. The expressions, that a number is a *multiple* of another, or a *divisor* of another number, are defined in the following manner.

"If we understand by a a given number and by n a given absolute number, and if we denote by b the number $a.n$ $(b = a.n)$, we call b the nth *multiple* of a, and the number a a divisor of b."

Using this terminology, we can enunciate the two theorems:

"The unit is a common divisor of all numbers in the number-system."

"Every number is a divisor of itself."

And we can immediately derive the theorem:

"If $a = b + c$, and if d is a divisor of two of the numbers, a, b, c, then it is a divisor also of the third number."

11. Now let a and b denote two different numbers of which the one is not a divisor of the other. In the Seventh Book of Euclid's *Elements*, it is shown how to find a common divisor of a and b which itself contains as a divisor every number which is a divisor both of a and of b. From this Euclidean process results a whole series of most important theorems in higher arithmetic and algebra. The method, in essentials, is as follows:

We will understand by a the preceding and by b the succeeding number. If we now form the successive multiples $a.1$, $a.2$, $a.3$, ..., we must arrive at a first one of these multiples of a which is a number coming after b, and such that the next preceding multiple of a—say $a.\beta$—precedes b.

Hence (§6), there must exist one and only one number c, such that

$$b = a.\beta + c.$$

This number c precedes the number a. If we now apply to c and a the same process that we have just applied to a and b, we obtain:

$$a = c.\gamma + d,$$

where γ is an absolute number, and d is a number in the same number-system as a, b, c, and precedes the number c.

By continued repetition of this process we obtain a sequence of equations:

$$b = a.\beta + c$$
$$a = c.\gamma + d$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$s = t.\tau + u$$

where a precedes b, c precedes a, d precedes c, and u precedes t. We arrive finally at an equation

$$t = u.\eta,$$

which asserts that the last number u is a divisor of the last but one number t.

Since now (§10) every divisor of a and b is also a divisor of c, and therefore of d, ..., s, t, u, and further, since u is a divisor of t, s, ..., d, c, b, a, we have obtained in u a number which is itself a divisor of both a and b and which, further contains as divisor every other common divisor of a and b. Also no other number can exist which possesses these properties; for a number which precedes u can never have the number u as divisor, and also u can contain no number as a divisor which succeeds u.

We can therefore enunciate the following theorem which forms the second proposition in the seventh book of Euclid's *Elements.*

"If a and b are two unequal numbers of the same number system, there exists always a common divisor of both numbers which itself contains as divisor every common divisor of the two numbers. There is only one number with this property, and it is obtained by the Euclidean process."

12. If a and b are absolute numbers, u is the *greatest common divisor* of a and b, and the Euclidean process has put us in possession of a means of finding this greatest common divisor.

"If this greatest common divisor is One, we call the two numbers *relatively prime numbers.*"

We are now in a position at once to derive the two following theorems:

"If a and b are relatively prime numbers, then $a.b$ is the least number which is a multiple both of a and of b."

"If d is the greatest common divisor of the numbers $a = d.a'$ and $b = d.b'$, then $d.a'.b'$ is the least number which is a multiple both of a and of b."

This number $d.a'.b'$ is called the *least common multiple* of the numbers a and b.

13. If, from this point, we were to penetrate further into the study of the addition and multiplication of whole numbers, we should arrive at the *Theory of Numbers,* a theory whose origin is found in the Euclidean process, and which, later, in the hands of Diophantus, of Fermat and Euler, Gauss, Lejeune Dirichlet, and Riemann and his followers and, finally, Minkowski, has become of such importance that a statement which is ascribed to Gauss, that the theory of numbers is the "Queen of Mathematics" in the same measure as Mathematics is the "Queen of the Sciences," gives us good reason for reflective thought.

In following this course we would have set out from the theory of prime numbers, "where by a prime number is to be understood an absolute number which has no divisors other than itself and One"; and then have begun with the proof of the fundamental theorem, "that every number is a product of prime numbers, and that, if two products of prime numbers represent the same number, then every prime number divisor of the one product occurs just as often in the other product."

CHAPTER II

The Fractional Number

1. If a denote a given number, we can, as we have seen, always form a second number b, which is a certain definite multiple of a. If, on the other hand, b is a given number, there does not necessarily exist any number, such that a certain multiple of it is equal to b. The equation, $x.n = b$, where x denotes a number in the same number system as b, and n denotes an absolute number, cannot be satisfied for all arbitrary values of n and b.

In order then to be able to retain the theorem that the above equation has *always*, i. e., for all arbitrary values of n and b, a solution, we must endow the unit with the attribute of *divisibility*, thus extending our number-system to include also numbers which are formed by *part* of the unit.

The only attribute which we have hitherto given to the unit is that of being embodied in the mind in the formation of number. We now postulate of it the further property, "that, however we may choose the absolute number n, the unit is always the nth multiple of another unit, which is uniquely determined by and with n. This latter unit we call the nth *fractional part* of the original unit, and denote it, if e denote the original unit, by $\frac{e}{n}$."

We can express this also as follows:

"We will assume of the unit e, that, for all values of the absolute number n, it is, in one and only one way, divisible into n mutually equivalent parts."

In every equality or inequality which involves the original unit e, and whatever the absolute number n may be, it must therefore be possible to replace the original unit by the nth multiple of the nth fractional part of itself $\left(e = \frac{e}{n} . n\right)$; and wherever the nth multiple of the nth fractional part of the original unit occurs, it must be possible to replace this nth multiple by the original unit $\left(\frac{e}{n} . n = e\right)$.

2. It is easy to see that every fractional part of the unit is itself divisible again into an arbitrarily large number of parts. To establish this we need to show only that, however we may choose the absolute numbers m and n, there is always a fractional part of the unit whose mth multiple is equal to $\frac{e}{n}$. The fractional part $\frac{e}{n . m}$ satisfies the equation $\frac{e}{n . m} . n . m = e$, and the fractional part $\frac{e}{n}$ satisfies the equation $e = \frac{e}{n} . n$.

But, since (Chap. I, §10),

$$\frac{e}{n . m} . n . m = \frac{e}{n . m} m . n$$

both the nth multiple of $\frac{e}{n . m} . m$ and also the nth multiple of $\frac{e}{n}$ are equal to e. But we have assumed that the nth fractional part of e, or that unit whose nth multiple is e, is uniquely determined, and therefore

$$\frac{e}{n . m} . m = \frac{e}{n};$$

and thus $\frac{e}{n . m}$ is the mth fractional part of $\frac{e}{n}$. In symbols

$$\frac{\frac{e}{n}}{m} = \frac{e}{n \cdot m}.$$

Since now $\frac{e}{n \cdot m}$ $\frac{e}{m \cdot n}$, we have the further theorem:

$$\text{“} \frac{\frac{e}{n}}{m} = \frac{\frac{e}{m}}{n} \text{”}$$

“Henceforward, in the number-system whose unit is e, we propose to call this unit the *fundamental unit* (Grundeinheit) of the system, and to include in the system both all numbers formed from the unit e, and also all numbers $\frac{e}{n} \cdot m$ which are formed from a fractional part of this unit but are not also multiples of this fundamental unit. The former will be called *integral numbers*, the latter *fractional numbers*.”

Hitherto the unit One has had only the one attribute of being embodied in the mind in the formation of number; we now give it the further attribute, “that it is divisible; and henceforward our *absolute number-system* will not only include *integral* but also *fractional numbers*.”

We have further:

$$\text{“}1 = \frac{1}{n} \cdot n \text{ and } \frac{1}{n} \cdot n = 1.\text{”}$$

Moreover we note

$$\text{“} \frac{e}{1} = e \text{ and } \frac{1}{1} = 1.\text{”}$$

“The number $\frac{e}{n} \cdot m$ may also be written $\frac{e \cdot m}{n}$, or, since $e \cdot m = b$, as $\frac{b}{n}$.” “This number $\frac{e}{n} \cdot m$ is the nth fractional

part of b"; for

$$\frac{b}{n}.n=\frac{e.m}{n}.n=\frac{e}{n}.m.n=\frac{e}{n}.n.m=e.m.$$

"The equation $x.n=b=e.m$, which served as our starting-point, is therefore *always* soluble, and has as its solution

$$x=\frac{b}{n}=\frac{e.m}{n}=\frac{e}{n}.m."$$

The number $\frac{b}{n}=\frac{e.m}{n}$ is called a *fraction*; $b=e.m$ is called the *numerator*, and the absolute integral number n is called the *denominator* of the fraction."

"When we speak merely of fractions without specifying that we are dealing with a number-system of fundamental unit e, it is to be understood that $e=1$, i. e., that we are dealing with numbers of the absolute number-system."

3. Since now $a=b$ no longer means that a is identically equal to b, but rather that a can be put into the form $\frac{e.m.p}{n.p}$ and b into the form $\frac{e.m.q}{n.q}$, where $p\neq q$, we see that, in order to compare two numbers with one another, it is necessary first to put them both into a common form. To effect this, one first puts each number into such a form that the absolute integral numbers in numerator and denominator are relatively prime. If, after this transformation, it appears that the one number is formed from the μth part and the other from the νth part of the unit, we must find λ, the least common multiple of μ and ν, and then transform each of the numbers into a multiple of $\frac{e}{\lambda}$. We can in this manner determine whether or not the two numbers are equal to one another, which number is to be regarded as the preceding and which as the succeeding; and, in case

$e = 1$, and the two numbers are unequal, which number is the greater and which the less.

We therefore obtain the results:

"If a, b, c belong to the same number-system, and if $a = b$ and $b = c$, then $a = c$.

If $a > b$ and $b > c$, then $a > c$.

If $a > b$ and $b = c$, then $a > c$.

If $a = b$ and $b > c$, then $a > c$.

4. We must next define what we are to understand by the sum of two numbers of our extended number-system. This definition must clearly read:

"By the sum of a and b we mean that number which results when both numbers are expressed as multiples of the same fractional part of the unit belonging to the system in question, and then summed as numbers whose *unit is that fractional part.*"

It is now seen without difficulty that "the sum $a + b$, defined in this manner, is unique and, in particular, independent of the choice of the fractional part with which as unit the two numbers a and b are expressed."

It is seen, moreover, that the Associative and Commutative Laws (Chap. I, §4) likewise retain their validity:

$$"a + (b + c) = (a + b) + c,$$
$$a + b = b + a";$$

and that our theorems of Chap. I, §§ 5, 6, also remain unaltered.

5. We have previously obtained the result (Chap. I, §7):

If $a_1, a_2, \ldots, a_n$ denote numbers of the same number-system and $m_1, m_2, \ldots, m_r$ denote absolute integral numbers, then

$$(a_1 + a_2 + \ldots + a_n)(m_1 + m_2 + \ldots + m_p)$$
$$= a_1 . m_1 + a_2 . m_1 + \ldots + a_n . m_1$$
$$+ a_1 . m_2 + a_2 . m_2 + \ldots + a_n . m_2$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$+ a_1 . m_p + a_2 . m_p + \ldots + a_n . m_p$$

In order to be able to retain this theorem also in the case when a and m are fractions, we must define what we mean by $\frac{e \cdot m}{n} \cdot \frac{p}{q}$, where m, n, p, q are absolute numbers.

We have

$$\frac{e \cdot m}{n} \cdot \frac{p}{q} = \left(\frac{e}{n} + \frac{e}{n} + \ldots + \frac{e}{n}\right)\left(\frac{1}{q} + \frac{1}{q} + \ldots + \frac{1}{q}\right)$$

where $\frac{e}{n}$ occurs m times in the first bracket and $\frac{1}{q}$ occurs p times in the second bracket.

If our theorem is to remain true we must therefore have.

$$\left(\frac{e}{n} + \frac{e}{n} + \ldots + \frac{e}{n}\right)\left(\frac{1}{q} + \frac{1}{q} + \ldots + \frac{1}{q}\right)$$
$$= \frac{e}{n} \cdot \frac{1}{q} + \frac{e}{n} \cdot \frac{1}{q} + \ldots + \frac{e}{n} \cdot \frac{1}{q}$$
$$+ \frac{e}{n} \cdot \frac{1}{q} + \frac{e}{n} \cdot \frac{1}{q} + \ldots + \frac{e}{n} \cdot \frac{1}{q}$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$+ \frac{e}{n} \cdot \frac{1}{q} + \frac{e}{n} \cdot \frac{1}{q} + \ldots + \frac{e}{n} \cdot \frac{1}{q}$$

where $\frac{e}{n} \cdot \frac{1}{q}$ occurs m times in each horizontal row and p times in each vertical column; and $\frac{e}{n} \cdot \frac{1}{q}$ denotes a number in the system of fundamental unit e, which it still remains to define.

If now $\frac{e}{n} \cdot \frac{1}{q}$ belong to this system, then

$$\left(\frac{e}{n} \cdot \frac{1}{q}\right) . q . n = \frac{e}{n} \cdot \frac{1}{q} + \frac{e}{n} \cdot \frac{1}{q} + \ldots\ldots + \frac{e}{n} \cdot \frac{1}{q}$$
$$+ \frac{e}{n} \cdot \frac{1}{q} + \frac{e}{n} \cdot \frac{1}{q} + \ldots\ldots + \frac{e}{n} \cdot \frac{1}{q}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ \frac{e}{n} \cdot \frac{1}{q} + \frac{e}{n} \cdot \frac{1}{q} + \ldots\ldots\ldots + \frac{e}{n} \cdot \frac{1}{q}$$

where $\frac{e}{n} \cdot \frac{1}{q}$ occurs q times in each horizontal row and the number of horizontal rows is n.

Now

$$\frac{e}{n} \cdot \frac{p}{q} = \frac{e}{n} \cdot \left(\frac{1}{q} + \ldots\ldots\ldots + \frac{1}{q}\right)$$

where $\frac{1}{q}$ occurs p times in the bracket; and accordingly

$$\frac{e}{n} \cdot \frac{q}{q} = \frac{e}{n} \cdot \left(\frac{1}{q} + \ldots\ldots\ldots + \frac{1}{q}\right)$$

where $\frac{1}{q}$ occurs q times in the bracket.

Hence,

$$\left(\frac{e}{n} \cdot \frac{1}{q}\right) . q . n = \frac{e}{n} \cdot \frac{q}{q} + \frac{e}{n} \cdot \frac{q}{q} + \ldots\ldots\ldots + \frac{e}{n} \cdot \frac{q}{q}$$
$$= \frac{e}{n} . 1 + \frac{e}{n} . 1 + \ldots\ldots\ldots + \frac{e}{n} . 1$$

where each term on the right-hand side of the sign of equality occurs n times.

We are therefore led to the result that if we are to retain our theorem in Chap. I, §7, without thereby leaving the

system of fundamental unit e, we must introduce the following definition:

$$\text{“}\frac{e}{n}\cdot\frac{1}{q}=\frac{e}{n.q}\text{”},$$

or, for $e=1$,

$$\text{“}\frac{1}{n}\cdot\frac{1}{q}=\frac{1}{n.q}\text{”}.$$

Starting from this definition, we obtain the theorem in question, and can therefore write:

$$\text{“}\frac{e.m}{n}\cdot\frac{p}{q}=\frac{e}{n}\cdot\frac{1}{q}.m.p=\frac{e}{n\ q}.m.p=\frac{e.m.p}{n.q}\text{.”}$$

We also see, without further proof, that the theorems in Chap. I, §10 and §11 remain true unaltered.

6. We have seen that the theory of fractional numbers arises from the attempt to retain the solubility of the equation

$$x.n=b=e.m$$

in the cases in which the absolute number n is not a divisor of the absolute number m.

This attempt, the material basis of which is the consciousness of our sense of the divisibility of certain material objects, goes back to the earliest childhood of the human race; although it was not until comparatively late that it was clothed in the abstract form that we have given to it here. Yet this form is, nevertheless, the adequate conceptual expression of what was, more or less unconsciously, sought after.

One of the most important tasks of mathematics, and of mathematical analysis in particular, is to show in complete clarity the ever-extending knowledge which men gradually acquire of objects and their relation to one another. It has

always been so, however far back into the history of thought we may go; it is so still in our time when it is a question of grasping in abstract form and of coördinating the many-sided and extensive content of new knowledge which constantly reaches us by way of the natural sciences; and so it will always remain as long as the world of our senses and the world of our thought stand in relation to each other.

7. Let a and b ($a < b$) be two numbers of our extended number-system (§2), which may be situated arbitrarily near to one another. However great the absolute number m may be taken to be, there are always m distinct numbers which succeed a and precede b. At first sight it might therefore appear as if the conception of infinity (Chap. I, §1) had, by the introduction of fractional numbers, gained an essential extension. That this, however, is not so may be seen as follows:

All absolute numbers may be expressed in the form $\frac{\lambda}{\mu}$. Now divide these numbers into groups, putting into the first group all numbers for which $\lambda + \mu = 2$ (therefore only the number 1), into the second group all numbers for which $\lambda + \mu = 3$ (i. e., 2 and ½), into the third group all numbers for which $\lambda + \mu = 4$, and which appear in neither of the preceding groups (therefore 3 and ⅓), into the mth group all numbers with $\lambda + \mu = m + 1$, which appear in none of the preceding groups, and so on. We see that in this manner the totality of all numbers is obtained. In the first group there is but one number, in the second two, in the third also two, in the mth group p_m numbers, and so on. Now correlate the numbers of our groups as follows with the numbers 1, 2, 3, ..., n, ... Correlate the number 1 with the number 1, the two numbers of the second group with the numbers 2 and 3, the two numbers of the third group with the two numbers 4 and 5.

Proceed in this manner, so that the p_m numbers of the mth group are made to correspond with the p_m numbers,

$$(p_1+p_2+\ldots+p_{m-1})+1,\ (p_1+p_2+\ldots\ldots+p_{m-1})+2,$$
$$\ldots\ldots\ldots\ldots,\ (p_1+p_2+\ldots\ldots+p_{m-1})+p_m.$$

In this way we have established a one-to-one correspondence between our whole universe of numbers and the sequence of numbers 1, 2, 3, ..., n, ...

In this process we have, for the first time, made use of the capacity of thought to retain simultaneously in consciousness two correlated objects. Without this capacity of thought there would be no mathematical science.

We have established that: *The conception of infinity is not extended by this introduction of fractional numbers.*

CHAPTER III

Groups of an Unlimited Number of Numbers. The General Absolute Number.

1. We have seen how all the integral and fractional numbers can be unambiguously correlated with the unlimited sequence of integral numbers $\mu = 1, 2, 3, \ldots\ldots$ in such a way that to every integral number there corresponds one and only one integral or fractional number a_μ; and conversely, to every integral or fractional number a_μ there corresponds one and only one integral number μ.

We now suppose that certain given numbers have been chosen from among the numbers a_μ. The aggregate of these numbers we will call a *group*, and denote it by (a); further, each particular number which belongs to the group will be called an *element* of the group.

Now it may happen that, when any number of elements, however great, is taken from the group (a), some elements still remain in the group. In this case we say that the group is the aggregate of an unlimited number of elements.

An example of such a group (a) which is the aggregate of an unlimited number of elements is afforded by the group whose elements are *all* the integral and fractional numbers; a second by the aggregate of all fractional numbers between two consecutive integral numbers; and yet a third by the aggregate of all numbers $\frac{1}{2^\nu}$, where ν assumes all integral values, or $\frac{1}{p^\nu}$, where p assumes all prime values and ν all integral values.

We will first simplify our discussion by the assumption that all numbers of the group are to be absolute numbers. Later we will deal with more general groups, for which this is not the case.

We begin by making, for the groups which we will now discuss, the following fundamental assumption:

Corresponding to every group there shall exist a number which is greater than any sum that can be formed from an arbitrarily large number of elements of the group.

We denote such a group by $((a))$.

In what follows we do not suppose given any previously defined correlation of the elements a with the numbers $\nu = 1, 2, 3, \ldots$, such as one would obtain if, for example, one regarded the numbers a as numbers a_{μ_ν}, chosen, in accordance with a given law, from the number-sequence a_μ.

Neither of the two groups formed by the totality of *all* integral and fractional numbers, or of all fractional numbers between two integral numbers, fulfils our fundamental assumption. For a sum of elements of such a group can always be formed which is greater than an arbitrary number given *a priori*, so that neither of the two groups is a group $((a))$. On the other hand, the group formed by the numbers $\frac{1}{2^\nu}$, where $\nu = 1, 2, 3, \ldots$, does satisfy the assumption; for, however large the number n may be, we have

$$\sum_{\nu=1}^{\nu=n} \frac{1}{2^\nu} = \frac{1\frac{1}{2^n}}{1\frac{1}{2}} \cdot \frac{1}{2} = 1 - \frac{1}{2^n} < 1.$$ [6]

The case must not be excluded in which certain elements of the group $((a))$ may be one and the same number; but, in consequence of our fundamental assumption, this can happen only for a finite number of elements and for any element only a finite number of times.

[6] For the sake of simplicity, and because the point is of no particular importance, we suppose the sign — to be known.

With the fundamental assumption which we have imposed on groups $((a))$, it will now be shown that in the theory of these groups we are on as firm ground as in the study of integral or of fractional numbers. It will be shown that these groups may be treated as numbers and are subject to the same rules of calculation as are integral and fractional numbers. It will appear further that their introduction involves an essential extension of the concept of number, whereby not only is given the conceptional significance of a problem which for thousands of years has baffled the efforts of the greatest thinkers, namely the fact that the side and the diagonal of a square, or again the perimeter and diameter of a circle, cannot be measured by a common measure; but also the way is opened from finite mathematics to the infinitesimal calculus with all its ramifications, whether they be higher mathematical analysis, differential or integral calculus, general function-theory or the theory of analytic functions or whatever other name we give them.

2. The first conceptions that we have to define for our groups are those of *equality* and *inequality*, of *greater than* and *less than*, as referring to two groups $((a))$ and $((b))$.

Equality exists between two numbers only if they can be transformed, after a finite number of operations, into the same number; but such a concept of equality can no longer be maintained as between the groups we are now discussing: the concept of *equality*, together with those of *greater than* and *less than*, must be given extended definitions.

We will use a terminology introduced by Weierstrass, and will say that a number μ is *covered* (German, *enthalten*) by the group $((a))$ if a number of elements can be selected from the group whose sum is greater than μ. By means of this terminology our fundamental assumption may be made to read:

A group which does not cover all the integral numbers is a group $((a))$.

We now define the concepts of *equality, greater than* and *less than,* as follows:

"The group $((a))$ is equal to the group $((b))$, $((a)) = ((b))$, $((b)) = ((a))$, if every number is covered by $((a))$ is covered by $((b))$, and conversely."

"On the other hand, the group $((a))$ is less than the group $((b))$, $((a)) < ((b))$, and the group $((b))$ is greater than the group $((a))$, $((b)) > ((a))$, if a number exists which is covered by $((b))$ but is not covered by $((a))$."

The statement that the group $((a))$ is unequal to the group $((b))$, $((a)) \neq ((b))$, then means that equality does not exist between the two groups, that *they are not equal to one another.*

We obtain an example of equality between two differently formed groups by comparing the group $((\frac{1}{2^\nu}))$, where $\nu = 1, 2, 3, \ldots\ldots$, with the group whose only element is the number 1. As we saw, however large the number n may be,

$$\sum_{\nu=1}^{\nu=n} \frac{1}{2^\nu} = 1 \frac{1}{2^n}$$

and it therefore follows that the group $((\frac{1}{2^\nu}))$ covers all proper fractions and that no other numbers are covered by the group. Since the same is true of the number 1, we have

$$((\frac{1}{2^\nu})) = 1.$$

3. It follows at once from the definition of equality between two groups that: "If the group contains only a limited or finite number of elements, it is equal to the sum of its elements."

And, further,

"If $((a)) = ((b))$ and $((b)) = ((c))$, then $((a)) = ((c))$."

"If $((a)) > ((b))$ and $((b)) >$ or $= ((c))$, then $((a)) > ((c))$."

"If $((a)) >$ or $= ((b))$ and $((b)) > ((c))$, then $((a)) > ((c))$."

4. Slight consideration shows also:

"Corresponding to every number δ that is covered by the group $((a))$ there exists an integral number n such that $$n.\delta < ((a)) < \text{or} = (n+1).\delta.$$"

For there always exists an integral number m so large that $m.\delta$ is greater than an *a priori* given number. If m is increased sufficiently we must therefore arrive finally at a number N ($N \nless m$) so large that N.δ is not covered by $((a))$. If now N be allowed to decrease by one unit at a time we must necessarily arrive at a last integral number $n+1$, such that $(n+1).\delta$ is not covered by $((a))$, whereas $n.\delta$ is covered by $((a))$, as was to be proved.

From the above it follows further: "If the group $((a))$ contain an unlimited number of elements, it is possible to remove from it so many of these elements (i. e., a sufficiently large number of the elements) that the group that remains is less than an arbitrary *a priori* given number."

For let δ be an arbitrarily small number covered by $((a))$. We have established the existence of an integral number n such that

$$n.\delta < ((a)) < \text{or} = ((n+1).\delta.$$

We can therefore always find an integral number m so large that

$$n.\delta < \sum_{\nu=1}^{\nu=m} a_\nu,$$

where $a_1, a_2, a_3, \ldots, a_m$ denote different elements of the

group $((a))$, together with a number $\delta_1 > \delta$ such that

$$n.\delta < n.\delta_1 < \sum_{\nu=1}^{\nu=m} a_\nu.$$

From this it follows that

$$n.\delta_1 + \sum_{\nu=m+1}^{\nu=m+p} a_\nu < \sum_{\nu=1}^{\nu=m+p} a_\nu,$$

where $a_{m+1}, a_{m+2}, \cdots, a_{m+p}$, are further elements of the group $((a))$.

Since now $((a)) < \text{or} = ((n+1).\delta$, we have

$$\sum_{\nu=1}^{\nu=m+p} a_\nu < (n+1).\delta = n.\delta + \delta = n.\delta_1 + \delta_2,$$

where δ_2 is a number less than δ.

Hence $$\sum_{\nu=m+1}^{\nu=m+p} a_\nu < \delta_2 < \delta;$$

and these inequalities are true, however large the number p may be taken to be. Thus

$$((a))^{m+1} < \text{or} = \delta_2 < \delta,$$

where $((a))^{m+1}$ denotes the aggregate of all those elements of a group that remain after m determinate elements, $a_1, a_2, \ldots, a_m$, have been removed; as was to be proved.

Conversely, we have the theorem:

"Let (a) be a group of numbers in the most general sense (see §1) and δ an arbitrarily small number; and let certain elements $a_1, a_2, \ldots, a_m$, be removed from (a). If the integral number m can be chosen so large that the group that remains does *not cover* δ, then (a) is a group $((a))$."

For if $a_{m+1}, a_{m+2}, \ldots, a_{m+p}$ are elements of the group that remains after the removal of $a_1, a_2, \ldots, a_m$, then

$$\sum_{\nu=m+1}^{\nu=m+p} < \delta,$$

however great the number p may be. So that we have, independently of the number p,

$$\sum_{\nu=1}^{\nu=m+p} a_\nu = \sum_{\nu=1}^{\nu=m} a_\nu + \sum_{\nu=m+1}^{\nu=m+p} a_\nu < \sum_{\nu=1}^{\nu=m} a_\nu + \delta;$$

and the group (a) is therefore a group $((a))$, as we had to prove.

5. A simple example will show that we have obtained in the groups $((a))$ something distinct from and more than those numbers which we already know. We choose the group $((1, \frac{1}{\lfloor\underline{\nu}}))$, formed by the aggregate of the numbers $1, \frac{1}{\lfloor\underline{\nu}}$, $(\lfloor\underline{\nu} = 1.2.3. \ldots .\nu)$, where $\nu = 1, 2, 3, \ldots\ldots$; of all groups $((a))$ in mathematical analysis which are not equal either to an integral or to a fractional number, this group, representing as it does the base e of the system of natural logarithms, was one of the earliest to be known and has been one of the most studied.

We have

$$\sum_{\nu=p}^{\nu=p+q} \frac{1}{\lfloor\underline{\nu}} = \frac{1}{\lfloor\underline{p}} \left(1 + \frac{1}{p+1} + \ldots. + \frac{1}{(p+1)\,(p+2)\ldots(p+q)}\right)$$

$$= \frac{1}{\lfloor\underline{p}} + \frac{1}{\lfloor\underline{p+1}} \left(1 + \frac{1}{p+2} \frac{1}{(p+2)\,(p+3)} + \ldots\ldots + \frac{1}{(p+2)\,(p+3)\ldots(p+q)}\right)$$

$$< \frac{1}{\underline{p}} + \frac{1}{\underline{p+1}}\left(1 + \frac{1}{p+2} + \frac{1}{(p+2)^2} + \cdots \frac{1}{(p+2)^{q-1}}\right)$$

$$\text{or} \frac{1}{\underline{p}} + \frac{1}{\underline{p+1}}\left(1 - \frac{1}{(p+2)^q} \Big/ \left(1 - \frac{1}{p+2}\right)\right)$$

$$< \frac{1}{\underline{p}} + \frac{1}{\underline{p}} \cdot \frac{p+2}{(p+1)^2}$$

$$\text{or} \frac{1}{\underline{p}} + \frac{1}{\underline{p}}\left(\frac{1}{p+1} + \frac{1}{(p+1)^2}\right).$$

If then q is an arbitrarily large number,

$$\sum_{\nu=1}^{\nu=1+q} \frac{1}{\underline{\nu}} < 1 + \frac{3}{4};$$

whence it follows that

$$2 > 1 + \sum_{\nu=1}^{\nu=1+q} \frac{1}{\underline{\nu}} < 2 + \frac{3}{4},$$

and that the group $(1, \frac{1}{\underline{\nu}})$ is a group $((a))$ or $((1, \frac{1}{\underline{\nu}}))$ with the property that

$$2 < ((1, \frac{1}{\underline{\nu}})) > \text{or} = 2 + \frac{3}{4},$$

and therefore cannot be equal to an integral number.

Nor can the group $((1, \frac{1}{\underline{\nu}}))$ be equal to a fractional number $\frac{p_1}{p}$ $(p > 1)$. We have shown in the preceding paragraph that if δ is a given but arbitrarily small number, we can always determine another number $\delta_1 < \delta$ and an integral number n, where δ_1 lies in the neighborhood of δ, and n is so large that

$$((1, \frac{1}{\underline{|\nu}}))^{n+1} < \delta_1 < \delta.$$

From this it follows that the sum of an arbitrarily large number of elements of the group $((1, \frac{1}{\underline{|\nu}}))$ is less than

$$1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \ldots\ldots + \frac{1}{\underline{|n}} + \delta,$$

so that if $\frac{p_1}{p} = ((1, \frac{1}{\underline{|\nu}}))$, then

$$\frac{p_1}{p} < 1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \ldots\ldots + \frac{1}{\underline{|n}} + \delta.$$

If then we write $n = p + q$, we have

$$\frac{p_1}{p} < 1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \ldots + \frac{1}{\underline{|p-1}} + \sum_{\nu=p}^{\nu=p+q} \frac{1}{\underline{|\nu}} + \delta$$

$$< 1 + \frac{1}{\underline{|1}} + \ldots\ldots + \frac{1}{\underline{|p-1}} + \frac{1}{\underline{|p}} \left(1 + \frac{1}{p+1} + \frac{1}{(p+1)^2}\right) + \delta$$

and hence

$$p_1 \cdot \underline{|p-1} < P + 1 + \frac{1}{p+1} + \frac{1}{(p+1)^2} + \delta . \underline{|p}$$

where P is an integral number determined in terms of p, and δ is arbitrarily small. Since then P and p $(p > 1)$ are given integral numbers, we must have

$$p_1 \cdot \underline{|p-1} < P + 1 + \frac{1}{p+1} + \frac{1}{(p+1)^2}.$$

But since, on the other hand, the group $((1, \frac{1}{\underline{|\nu}}))$ covers the number $1 + \sum_{\nu=1}^{\nu=n} \frac{1}{\underline{|\nu}}$, from the equation $\frac{p_1}{p} = ((1, \frac{1}{\underline{|\nu}}))$ we

should deduce the inequality

$$\frac{p_1}{p} > 1 + \sum_{\nu=1}^{\nu=n} \frac{1}{\underline{|\nu}} \text{ or } 1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \ldots + \frac{1}{\underline{|p-1}} + \sum_{\nu=p}^{\nu=n} \frac{1}{\underline{|\nu}}$$

$$> 1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \ldots + \frac{1}{\underline{|p-1}} + \frac{1}{\underline{|p}};$$

and therefore $p_1 . \underline{|p-1} > P + 1$.

Thus,

$$P + 1 < p_1 . \underline{|p-1} < P + 1 + \frac{1}{p+1} + \frac{1}{(p+1)^2}$$

Since the numbers P and $p_1 . \underline{|p-1}$ are integral numbers and $\frac{1}{p+1} + \frac{1}{(p+1)^2}$ is a proper fraction, the above inequalities would assert that a certain integral number lay between a second integral number and the sum of the latter integral number and a proper fraction. Since this is impossible, and since also $((1, \frac{1}{\underline{|\nu}}))$ is not equal to an integral number, we have obtained in the group

$$e = ((1, \frac{1}{\underline{|\nu}}))$$

a group $((a))$ which is equal neither to an integral nor to a fractional number; as was to be proved.

"The concept of group is more general than the concept of integral and fractional numbers. Groups exist which are neither equal to an integral number nor equal to a fractional number."

6. This theorem is again only a special case of the more general theorem: "Let A and B $(A < B)$ be two arbitrary numbers, which are therefore arbitrarily near together. A group $((a))$ always exists which is equal neither to an

integral nor to a fractional number and which lies between A and B; i. e., such that $A < ((a)) < B$."[7]

This theorem forms part of a chain of deductions which Weierstrass introduced in order to establish another theorem, which, again, forms the indispensable foundation of every firmly-established theory of functions. This latter theorem, of which Weierstrass gave both the first clear enunciation and the first rigorous proof, reads:

"Let A and B be two numbers or groups, of which $A < B$; and suppose that in the interval $[A < B]$ an unlimited number of distinct numbers or groups $((a))$ are given. Then the interval $[A < B]$ contains at least one group $((c))$ which is a *limiting-value*[8] for the groups $((a))$."

"By a limiting-value $((c))$ we mean a group such that, if it be enclosed between two numbers, which may be taken arbitrarily near to one another, there always lie between these two numbers an unlimited number of the given groups $((a))$."

As examples we note:

$A = 1 \qquad B = 2 \qquad ((a)) = 1 + \frac{1}{2^n}$

$n = 1, 2, 3, \ldots . \qquad ((c)) = 1$

$A = 2 \qquad B = 3$

$$((a)) = 1 + \sum_{\nu=1}^{\nu=n} \frac{1}{\lfloor \nu};$$

$n = 1, 2, 3, \ldots ; \qquad ((c)) =$ the group e (Cf. §5).

[7] See George Cantor, *Uber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen.* Journal f. d. reine und angew Math., Vol. 77, §2, p. 260. Also *Ueber unendliche lineare Punktmannigfaltigkeiten,* Math. Ann., Vol. 15, p. 5. Also *Sur une propriété du système de tous les nombres algébriques réels,* Acta Mathematica, Vol. 2, §2, p. 308.

Henri Poincaré, *Ueber transfinite Zahlen,* p. 45; *Sechs Vorträge, aus d. reinen Mathematik und mathem. Physik,* Fifth Lecture.

[8] German, *Grengstelle.*

The Weierstrassian chain of deduction includes also Georg Cantor's famous theorem:

"If $((a_1))$, $((a_2))$, $((a_3))$, ... are an unlimited number of $((a))$ groups, of which no two are the same, which are in one-one correspondence with the integral numbers 1, 2, 3, ..., a new group $((c))$, distinct from all the given groups $((a))$, can always be formed satisfying the condition $A < ((c)) < B$, where A and B are two arbitrary numbers, arbitrarily close together." [9]

G. MITTAG-LEFFLER.

DJURSHOLM.

[9] Authorized English translation by B. M. Wilson.

SCIENCE AND MYSTICISM

I looked, and behold! the cloud wrote!

I

THE sun was sinking, a molten ball, into the depths of an incandescent sea, the sky was all aglow with a lambent light whose very faintness made its intensity the more profound. Afloat on the shining amber was a long rose-gold cloud, almost a perfect feather. The barbs curled gracefully, long threads of mist-like wires of flame. The quill pointed straight into the curtain of amber light, ready to write. Higher up the evening star was vivid in white radiance, too fluid, too soft, too dazzling, to be called silver. The breezes ceased whispering, and in the hush, everything seemed expectant of the message that would appear in living letters across the infinite depths, glowing with meaning. The sun sank, the hues deepened, the feather floated gently along, writing the message as it moved.

To the physical eye the message remained a secret. Though the glowing light passed the crystalline lens, impinging upon the retina, not all the alchemy of that sensitive plate could transmute the words into visible forms. The optic nerve was stirred to pass the message on to the occiptal lobes, but even in their shadowy depths the vibrations did not dissolve into anything that could be read. The physicist talks learnedly about radiation waves with different frequencies, about the absorption spectra of layers of humid air, about varying densities, about transmission of reds, oranges, yellows, seen against a sky whose blue

is due to the particles that float below, but in none of his discourse does he glimpse that great message written by the golden cloud. The astronomer too talks of the beautiful planet of the evening, either wrapped in a dense layer of radiant cloud, or exposing the brilliant surface of an everlasting desert, thus appearing in its dazzling white. But in none of his telescopes does he see the words of living fire. No interferometer can catch the magic sentence, no spectroscope spell its jewelled letters. They deal with nothing but appearances, with phenomena, not with reality. What they try to do is to tabulate the uniformities in phenomena, and from these tabulations to guess what may be the reality.

The biologist and psychologist also talk at length about a highly complex organism whose receptors are easily responsive to stimuli. They see a mechanism in such a delicate equilibrium that even a wave of radiation can disturb the balance, and over the quivering threads woven into this network run currents which flow through the synapses, spreading into the multiform channels, and exciting other similar responses, like a wonderful system of silver bells, all set ringing by a single feather touch. They talk of imagery floating in the cells of the nervous system, of organized sets of cells; of intricate nets, coöperating with all the precision of a well-drilled army, yet as fluid as the wisps of evening mist that float in a river valley; of outgoing currents that set into contraction various muscles and systems of muscles, tightening the throat with a wave of emotion, making the heart beat with the thrill of beauty. And that is all! In all the marvellous web of human life as it appears in the body, delicate in response beyond all explication, there is nowhere to be read the words written across the amber sky. It is as if they should try to explain why the recipient of a long distance telephone message blanches and drops dead, by undertaking to describe the telephone

apparatus, the vibrating diaphragms of the transmitter and of the receiver, the complicated electric waves that oscillate in the copper wires. The significance of the message, the reality behind it, in it, are never imprisoned in this net, however fine its meshes. They must be apprehended in some other way. It may be true that the investigation of phenomena when pushed far enough, when examined under the most vivid light the intellect can cast, will show the faint shadow of an unseen presence which is implied indeed by the very phenomena, just as microscopic deflections of rays of light would show the existence of unseen and unsuspected masses, or perhaps unimaginable curvatures in space. Even though the flow of phenomena is a moving picture show, and analysis shows nothing but a succession of snapshots, yet we are forced ultimately to conclude that after all the pictures are but the shadows of something else which is real, and living, and active.

Mysticism in its broad sense means the awareness of the invisible world, the world of beauty, of truth, of ideals, of religion. It is appreciated by the intuition, and is expressed in works of art, in music, in religious ceremonies, in all the myriad manifestations of what is called the spiritual life. In the narrower sense, mysticism usually means the more or less vivid awareness of God in the universe. There have been mystics at all times, in all countries, in all religions. Hindu, Buddhist, Egyptian, Greek, Alexandrian, Moslem, Christian, have all found more or less fully, a way of life, a path, a curve of development which leads to attainment. In the Sacred books of Hermes Trismegistus are the words: "Comprehend clearly that this sensible world is enfolded as in a garment by the supernal world." "Celestial order reigns over terrestrial order; all that is said and done upon earth has its origin in the heights, from which all issues are dispensed with measure and equilibrium; nor is there anything which does not emanate from above and

return thither." (*Virgin of the World.*) Swedenborg said: "It is to be observed that the natural world exists and subsists from the spiritual world just as effect exists from its efficient cause." In Plato's Shadow-watchers we have a statement of the enduring archetypes behind the transient phenomena of the sensuous world. In the present time Croce tells us: "The spirit is history, makes history, and is itself also the result of all preceding history, since becoming is the essential reality, the creative formula of that life in which we find ourselves immersed." Mysticism is not the fanciful daydream of a poet, nor the hazy phantasy of a hysteric in a trance, but the clear vision of those who see reality face to face. The terms in which the vision is described matter little, so that they are intelligible to the sympathy of those who listen. It may be the Hindoo, "Om Mani Padme Houm!" or the Moslem "Allahou Akbar!" or the Christian: "God is Love!" It may be St. Catherine and St. Teresa in their Catholic phrases, or St. Jeanne D'Arc in the language of the angels who spoke to her, or Fox and Wesley in the quaint language of the early Bible. Thomas à Kempis made an attempt, modern at his time, to describe the life of the spirit. In our present day we find many trying to do the same thing, from such attempts as Trine's, "In Tune with the Infinite," and Anna Payson Call's, "Power Through Repose," through the poetry of Browning, and others, up to the late book of Evelyn Underhill: "The Life of the Spirit and the Life of Today." She says: "The artist, the poet, every one who looks with awe and rapture on created things, acknowledges in this act the Immanent God." Again: "Here is our little planet, chiefly occupied, to our view, in rushing around the sun; but perhaps found from another angle to fill quite another part in the cosmic scheme. And of this apparently unimportant speck, wandering among systems of suns, the appearance of life and its slow development and ever-in-

creasing sensitization; the emerging of pain and pleasure; and presently man with his growing capacity for self-affirmation and self-sacrifice, for rapture and for grief. Love with its unearthly happiness, unmeasured devotion, and limitless pain; all the ecstacy, all the anguish that we extract from the rhythm of life and death. It is much, really, for one little planet to bring to birth. And presently another music, which some—not many, perhaps yet, in comparison with its population—are able to hear. The music of a more inward life, a sort of fugue in which the eternal and temporal are mingled; and here and there, some already who respond to it. Those who hear it would not all agree as to the nature of the melody; but all would agree that it is something different in kind from the rhythm of life and death. And in their surrender to this—to which, as they feel sure, the physical order is really keeping time —they taste a larger life; more universal, more divine. As Plotinus said, they are looking at the Conductor in the midst; and, keeping time with Him, find the fulfillment both of their striving and of their peace."

II

There is much discussion, fruitless even if not heated, over the results of Science, and those of Mysticism. On the one hand, Science has been bitterly assailed as teaching materialism, determinism; of asserting the non-existence of mind, of spirit, of immortality, of beauty, of God; and on the other hand mystics have been called fanatics, visionaries, dreamers, full of superstition, and the illusions of an emotional nature. If we consider what each is doing, however, we find readily enough that they are complementary and should assist each other.

Science, for instance, is condemned for not doing what it has never undertaken to do. We do not find Science considering beauty. For it, the fireflies among the cherry blossoms of Nippon are only entomological specimens circulating among sterile flowers. The forest of glittering glass made by the hand of the glaze storm is no magic wood, but merely the result of vortices and temperature gradients in the atmosphere. The little waves that snip-snap on the beach are not telling stories of the palm-shaded isles of the far-away, but are merely setting up longitudinal condensations and rarefactions in the air through the impulses of a roughly rhythmic motion. The quivering air in diaphanous silk of amethyst, topaz, and beryl, over the Painted Desert, is resolved into floating dust particles and irregular heat currents. The ruby and emerald streamers of the Aurora conceal no mystery behind their fluttering curtains, for they are merely vibrations due to ionized atoms in the upper air. Spring does not dance over the fields and through the woods strewing pink and azure flowers, nor does Autumn gild the leaves with gold and bronze ere she floats away in blue haze, for both Spring and Autumn are but results of the inclination of the axis of rotation of the earth. The beginning of life is no mystery, for it is due to the arrangements of chromosomes and the particles they carry like beads on a string, and the coming of death to darken with his finger everything into a somber solemnity, is merely the cessation of chemical reactions. Science does not thrill at the Yosemite, stand in awe before the Grand Canyon, melt in the mockingbird's rhapsody, nor feel the catch in the throat produced by a desert sunset. These things are not for microscopes, galvanometers, or other measuring apparatus.

Science does not undertake to discover purposes, if there be any, in the universe. Carbon, hydrogen, oxygen, nitrogen, as viewed by it, are not for the purpose of enabling

plants to grow, nor does wheat and corn exist, in order that cattle and horses may be fed, nor do fish, fowl, and flesh exist that man may have a varied diet. It is true these things happen, but the investigations of Science are not in the direction of purpose. The bee does not search for nectar that flowers may be pollenized, nor bacteria develop to nitrogenize the soil. The axis of the earth was not tipped so as to produce a varied climate over a large part of the surface, nor was the pressure of the air, the amount of moisture, and the temperature so adjusted as to make the planet the home of a prodigious amount of diversified and active life. On the other hand, according to the second principle of thermodynamics the universe is running down, and when all energy is uniformly distributed, all motion and all life will cease, and a universal stagnation will exist. Science is not concerned with the questions: Whence? Why? Whither? Wherefore? Man is of no more importance than a streptococcus. Evolution is merely an account of the forms that have survived in the conflict of opposing forces, complex and intricate beyond all description; so the appearance of a new creature is no miracle, the disappearance or extinction of a species or a genus is no tragedy. The goal of Science is not the discovery of purposes but the attainment of an intelligent knowledge as to the construction of the universe, large and small. Whether the earth carry smiles or tears is all one to Science. It views everything unemotionally, impersonally, placid as the Sphinx before the shifting desert sands, the petty march of Napoleon's armies, the precession of the equinoxes.

Science does not study, nor even admit spontaneity. Nothing it considers is looked upon as undetermined. The orbit of the humming-bird, though tangled, is nevertheless as determined as that of the moon. The butterfly's poise on the rim of a buttercup, the song of the thrush, the scream of the mountain-lion, the track of the hurrying ant,

the wild duck's trail in the night, the migrations of man, the pioneer building his cabin, the riveter swinging on a steel girder, the aviator's roar, the battling legions in Europe,—all were written down millions upon millions of years ago in the whirling electrons and clustered atoms, in the fields of force and energy. Though politician and statesman scheme for their own or their nation's advancement, their scheming is only part of the great cosmic machine which cares naught for greatness. Though missionaries pour out their lives in heathen lands, they have no claim to merit, for what they do was written as inevitably as the path of the earth that hurries to meet a meteor stream. In science chance can play no part, freedom has no wings. The stresses in the ether or in space force one to kill, and one to execute, one to weep that another may sing, and neither could have done otherwise. The machine might even go backwards and there would be "progress" either way. What is to come is written in today's scroll, and it contains likewise what was. Calculating an eclipse a thousand years ago or a thousand years hence is a problem of arithmetic. So too calculating the path of Jeanne D'Arc in the days of her youth and glory is equally possible.

True, Science may have to abandon this position, for the forces of statistical inquiry, are ever advancing, and taking as their domain the territory of determinism. But every scientist hopes today to be able sometime to state his subject as definitely as do the physicist and chemist, to reduce the universe to a few elements in terms of which all may be calculated, and predicted.

Science is impersonal. It respects no authority, and its proudest investigator, as well as its humblest, must attain results which anyone else may attain by repeating the work. Consequently, it has no interest in individual experiences. The most intense joy, the most intense suffering, of one person is no matter of scientific concern. It studies only

the common elements in the experiences of many persons. What is not common, cannot be repeated over and over, not impersonal, is not for science. Everything must be subject to observation, to experimentation, to varying conditions. Indeed, personality itself becomes for science only a coherent group of reactions, not very different in character from any other group, as for instance a cell, or a molecule. The solar system could be called a personality, or the whole universe. The human personality, is merely the sum total of the human body with all its performances.

These statements may or may not be an indictment of science. One does not expect a painter to be a chemist of pigments, nor a teleologist to write on the theory of electricity, nor a theologian to explain relativity. If they have tried such tasks they have merely shown that they did not know as well as does the scientist what they were entitled to discuss. And one may find beauty in the results of science, even though the scientist is not looking for beauty, and sometimes seems to destroy beauty. One may find that the results of science do point to design, but that is not what he is examining. One may discover that the results of science indicate a certain amount of indetermination, slack in the machine, points at which choice must enter, but the scientist is not able to admit such a conclusion. For these omissions he may not be indicted, but when the scientist says to the world, "Behold, I have proved that everything is but whirling dust, and to dust returns; I have demonstrated that Beauty is merely a nervous reaction, that Truth is merely a statement of what happens, that there is no ideality nor spontaneity, that

> The worldly Hope men set their Hearts upon
> Turns ashes—or it prospers; and anon,
> Like Snow upon the Desert's dusty face,
> Lighting a little Hour or two—is gone!"

—when Science or scientist says this, then we must indict them.

What then is it that Science spends her long and patient hours studying? What do men work for years to gain, sometimes to find that there is nothing to gain? First of all, Science is engaged in finding out the *uniformities* that exist in phenomena. We may conceive a universe in which scarcely anything would ever be repeated; in which today water would boil at fifty degrees, and tomorrow would boil at three hundred degrees; today we could plant wheat and next summer gather cotton; that from thorns we would garner grapes, and figs from thistles. Such a universe might be extraordinarily beautiful, might be designed for the most noble purposes, and its inhabitants might be most lovable creatures. Spontaneity would revel as caprice, freedom would appear as chance. Nothing would be stable, and law could not exist. But the scientist asserts that the universe as we know it is full of law, he finds all sorts of uniformities, even in opposing situations. Stars and planets swing on their age-long paths under inflexible law, combining in systems that remain stable. Protons and electrons too obey law, even though they explode and seem quite unstable. Single cells and huge animals exhibit the control of law, structures minute or gigantic are built by a few forces under simple laws. The scientist is hunting for these *invariants* of the universe and his quest has been highly successful. He has become intoxicated with his conquests. The only question which comes to sit like a ghost at the feast, is whether he *finds law,* or whether he *imposes* law. Does he but analyze out of his thinking a characteristic he himself has furnished? Says Eddington (*Time, Space, and Gravitation*): "We have found a strange footprint on the shores of the unknown. We have devised profound theories, one after another to account for its origin. At last, we have succeeded in reconstruct-

ing the creature that made the footprint, and Lo! it is our own."

Science seeks *simplicity*. One of its chief aims is to find the few irreducible elements in terms of which the whole universe may be stated. When Newton wrote in a single law the motion of all the planets, it was a tremendous gain, and the simple notion of gravitation was the base for the magnificent structure of celestial mechanics. Science has succeeded beyond the dreams of the most sanguine investigator of a century ago, in finding a few simple elements for all the intricate confusion of the universe. Electrons and protons make up all matter, whether in one atom or in the titanic Betelgeuse; out of these are built atoms, molecules, colloidal units, cells, organisms, planets, suns, galaxies, and the universe. What a triumph! Even these are about to be reduced to energy in motion, so that one single substance would constitute the entire universe. But an unexpected result of this success grins like a gargoyle on a cathedral tower: the indestructibility of matter seems to be no longer a demonstrated fact, for the Bohr atom suggests that electrons may vanish out of existence, and Eddington's work on the stars suggests that every giant sun is an electron factory. Indeed, the law of the conservation of energy has been merely a principle, not a law, for some time, and now the indestructibility of matter is in question. The biologist has had much success in reducing his elements to a small number, he has even tried to state them in the terms of the physicist and chemist, but he has found a stubborn inadequacy in them. He has not yet reduced life, even as he views it, to physics. The psychologist too has tried his hand, and while he has reduced his data very much, he also finds that biology, physics and chemistry do not suffice to state his facts. As the result of all these simplifications, indeed, we find that we need not be bewildered by the multitude of diversities in phenomena,

but can maintain our poise, and understand the universe in a relatively few simple terms. The simplicity has been in the elements, at the expense of the configurations, since these have become immensely more intricate.

After deciding what are the irreducible elements of the universe and what are the laws they are subject to, Science undertakes to construct what it calls a *description* of the universe. Since there has been abstraction all along the line, however, an ignoring of certain component parts, the description remains but the description of certain aspects of the world. It is a sort of conceptual replica of the material universe, and is by no means exhaustive. The chemist for instance puts together his electrons and protons in geometrical configurations to represent atoms, these by the interconnection of valence electrons are built into molecules. In this manner he proceeds to construct in thought the objects that he is dealing with. The biologist spins threads in a cell, weaves cells into organs, and organs into organisms, and is able thus to picture to himself the objects he is studying. The psychologist incorporates mental processes into more and more complex integrations, arriving at a network which enables him to visualize his material. It is a very interesting game, the only drawback being that occasionally the scientist runs across pieces that will not fit, and he has to take the puzzle apart and start over. When Madame Curie discovered radium, the bombardment it opened upon the foundations of physics has almost destroyed them. But the new structure will no doubt last for a long time. The study of nebulae has swept away the fire-mist of former days, and made a model of the universe of stars and worlds, in their courses, which resembles a swirl of snowflakes, that gather material as they fly, haphazard almost in their growth. The collisions, and the near passage of huge masses close by each other's paths, has done the rest.

"A moment's Halt—a momentary taste
Of Being from the Well amid the Waste—
And Lo!—the phantom Caravan has reach'd
The Nothing it set out from—Oh, make Haste!"

Science has given us then at least empirical *laws,* simple *elements, models* for the objects we study, which serve to exhibit the relations in phenomena. Its progress has been great in this stupendous task, and the scientist has a right to be proud of what he has accomplished. He is Martha who has set the house in order. But he should not then be censorious of Mary, the dreamer, the mystic, who is looking behind phenomena for reality. She is intent upon solving the master-knot of human fate. Indeed for him to assert the reign of universal law, is to become a mystic. For this is a generalization, which can never be substantiated, it can only be made more or less probable. And to assert that one *sees* universal law is to assert that he can penetrate behind the veil, can watch the palpitating heart of the proton, can go through the wall of the electron, can surround the entire universe. Though most observed phenomena yield up the unbreaking bands of law, it is not possible to go further without becoming a mystic. It is really the perception, dim and confused, of God in the universe, with a character of stability, the same yesterday, today, and forever. The confusion comes in ascribing to this stability an inflexibility which permits no oscillations. Even mechanics shows that remarkably stable systems permit oscillations about positions of stability which are not always even small. And the whole of modern physics is dependent upon a discreteness in structure, which makes continuity impossible, and upon discontinuous motion, which is nearly unthinkable. The scientist must recognize then that despite his attempt to avoid mysticism he has ultimately exercised the mystical vision, though somewhat out

of focus. We have evolved in a universe stretched upon the steel wires of law, and we consequently find comfort and satisfaction in the permanence due to law, but what we might be in a universe where there was no law, we do not know. It might be a preferable condition. At least we cannot assert that miracles are impossible in our universe. We are not even sure that new laws do not come into existence now and then.

The notion of simplicity in the fundamental elements of the Universe is also a vague result of mysticism that has been hindered in its vision. For the existence of such elements as electrons rests upon purely inferential evidence, and is dependent upon hypotheses, which, if they were changed for others, might make the conclusions untenable. But to go further and consider everything as made of energy in motion, when energy itself can scarcely be conceived as a substance, is certainly to venture into the territory of the mystic. It is in fact the mystical notion of Unity showing up, in a rather disguised form it is true, but yet discernable. In the end we must admit that these entities called elements are to a certain extent creations of the scientist's mind. There have been many such created and laid away in the museums of science. Phlogiston has gone its way; the ether has disappeared, which was once possessed of a rigidity far beyond that of steel; force is nowadays only a name for a mathematical entity, like mass, action, potential and field. The relativity theory has shown that other names which stood for actual objects represented only creations of the mind. Indeed, the conclusion is one scarcely to be expected from science, but nevertheless true, that most of its objects are *creations*. The important fact is then the mystical one that mind is a creative agency. This fact certainly demands going behind phenomena to see reality back of them. If a list of terms from the sciences is examined there will be surprise at the large num-

ber that turn out to be creations made to explain phenomena and their relations. When to these we add the list of "tendencies," "dispositions," "inclinations," and the like, we may well conclude that the simplification that science reaches is a simplification in thinking not in things. Created entities, however, are no less "real" than those that may be supposed to be inherent in the phenomena, for in the end they are in the mind equally, and whatever reality one possesses the other has the same kind belonging to it.

The success of science in constructing a model, conceptual or otherwise, of the world also has a mystical result, for it shows that if the model is accurate, then the human mind is capable of comprehending the material universe, of analyzing it, and of arriving at its essential characters; and if the model is not accurate, then it shows that the mind is able to create things for itself which are different from the visible universe, in other terms to transcend its sense-data. Since this creative or spontaneous ability does not exist in the world of science, it is a mystical result. In either case then science is confronted with the *creature which makes science.*

There are other results of science which may also be shown to come from a source which is mystical, that is, a source which wells up from the invisible, the intangible world. It thus becomes apparent that although science does not investigate the flower-strewn fields of mysticism, yet in its own domain bloom unsuspected blossoms, and when its field is seen from the heights, so that it may be viewed as laid out, like a beautiful pattern below, its paths all trend toward the slopes, its hills are flooded with sunlight, its vales are musical with real songs. Science indeed assures us that the winds of destiny blow, heedless of our desires, but its universe though pitiless is just, destitute of purposes but stable, the manifestation of power, which is not capricious, infinite in intricacy, yet seen in the mirror

of thought it is intelligible. It finds in spite of itself the mystic notions of universal law, unity, and simplicity. It can therefore say to its sister, Mysticism, "I give thee these rich gifts, guard them well on thy way to the Heights. Find the Master

> "Whose secret Presence, through Creation's veins
> Running quicksilver-like eludes your pains;
> Taking all shapes from Mah to Mahi; and
> They change and perish all—but He remains."

III

There are two ways in which we adjust ourselves to the world in which we live. One of these is the intellectual study of the data furnished by the senses. This always involves abstraction. If we study an animal, we consider separately the anatomy, the physiology, the behavior. As a unit the animal is not the subject of study by these processes. Even when all the different abstract studies combine their results, we have only the sum of the aspects from the different points of view, not the living creature. The other way we adjust ourselves to the world is by what Bergson calls the Sympathy. We might, by using our terms carefully, speak of it as the Spiritual View. It is what is called sometimes the Intuition, which must be understood in a wide sense. The word sympathy is close enough for our purposes at present. This means that to understand a work of art we must have a "feeling for art," to understand a mathematical investigation we must have a "feeling for mathematics." This mode of apprehension of the universe is not reducible to terms of the intellect. It is possible for instance to become an intellectual critic of

music of high order, and yet to fail to have any real appreciation of the "soul of the music." Mysticism is the result of the latter mode of viewing the world, of seeing through the world. No amount of the scientific method of investigation can take its place. They do not consider the same things. Scientific investigation considers sense-data given in observation and experiment. Mysticism does not consider sense-data at all. It considers the data of an immediate awareness of things that are not given through the senses. Science has emphasized for so long the use of sense-data that it has become well-nigh impossible for a scientist to form any idea at all of other data. There are such however and these we will proceed to consider. We will have a little difficulty owing to the fact that the spiritual vocabulary is rather meager by the side of the rich resources of the intellectual vocabulary, but we can get along.

We may discriminate four forms of mysticism, among others: Philosophical, Mathematical, Artistic, and Religious. The first is sometimes called metaphysical, but this term is usually reserved for a different meaning. Plato was the great philosophical mystic, though there were many others. In opposition to him was Aristotle, who might be called the philosophical scientist. Thinkers are often classified as Aristotelian or Platonic in their points of view. Philosophy for Plato was a mode of life, and he undertook to consider life first hand. It is not remarkable from his standpoint that he placed mathematics in so high a position, for it was too, as he saw it, a subject for mystic, or intuitional perception. Life was to be maintained in harmony with certain great principles, or Ideas, which were directly perceived, at least when they were elicited by the skillful tongue of Socrates. We may go back twenty-five centuries, however, and find the great mystic, Pythagoras, to whom all the universe was an immense symphony. We may find Heracleitos watching the mountain torrent,

the drifting clouds, the dancing flames, and sensing the everlasting flux of the universe. We may find Parmenides impressed by the mountains, the ever-recurring sunset, the velvety sky covered with glittering gems, so that he saw the eternal character in the universe. Many philosophers have been mystics, though many have not, for the analytic, critical philosopher is not a mystic usually. Spirit is elusive of anatomical investigation, slipping out at the important moment and leaving that which is dead behind.

Mathematicians have not been called mystics often, but we do find an occasional characterization of the profoundest mathematical work as "mystical." There are those to whom the fourth dimension, non-Euclidean geometry, non-intuitional functions, hypernumbers, etc., are mere empty symbols. This is the point of view taken, for instance, by Rignano in his late book on the "Psychology of Reasoning." It has been asserted that mathematicians cannot make good psychologists (and considering the meaning of "psychologist" in such connection, it is true, fortunately, for the mathematician who would be the dupe of his senses, would be a sorry specimen), the reason being that they deal with ideal constructions, with a world which they have created, in short with data that do not come in the contents of "sense-experience." It is true the mathematician does not depend upon experiment, that he has no interest in the pragmatic testing of his results. His triangles are not made of chalk, his numbers are not mere "incorporations of operations upon integers" which themselves come out of the counting of material objects. Space does not permit the complete discussion of this topic here, but it may be found elsewhere. The mathematician accepts the appelation "mystic" however when it means that he is able to view the invisible, to handle the intangible, to perceive relations directly, to thrill with the beauty of pure form, to be swayed by the music of unalloyed harmony, to bow before sym-

metries never dreamed of by the painter, to stand in awe before the profusion of eternal worlds with which he is acquainted. The world of mathematics is the fairyland of existence, where indeed the spirit is free and can make frost-flowers of phantasy blossom in exquisite delicacy, where gauzy-winged creatures flit unhampered by the leaden weights of energy and matter, where in fact all the earthly characters of "experience" have dropped away like Cinderella's rags, where Prospero waves his wand and dream-castles come true. But his worlds do not vanish at the rising of the sun, for in the most brilliant light the intellectual devotees have ever been able to pour upon them, the most searching, critical, investigation any scientist has ever devised, they stand, eternal, beautiful, sublime, spotless in purity, the unanswerable argument that we can comprehend the invisible, the intangible, that which transcends the data of the senses. They validate the claims of the mystics of all kinds, philosophical, artistic, or religious. For who would destroy mysticism, must first destroy mathematics.

Every artist—using the term in the wide sense—is a mystic. For every artist is endeavoring to express the invisible, the intangible, to put into material form in some way the vision he has through an inner sense. When he uses marble he imitates natural forms but with a magician's touch, to incarnate the lovely ideal creatures of his spiritual sight. If he is unusually skillful they almost breathe and step down from their pedestal. When the artist uses color in imitating the sunset, the sea, glowing canyon walls, or the tinted human form, he is trying to make his canvas palpitate with life, to glow with the spiritual color, to catch Ariel's blithesome beauty. When he uses thin air, then indeed, in his imitation of the rhythm, the Dance of Life, he is almost able to show us the surging activity of that dauntless creature beating against its barriers.

"But here is the finger of God, a flash of the will that can,
Existent behind all laws, that made them and, lo! they are!
And I know not if save in this, such gift be allowed to man,
That out of three sounds he frame, not a fourth sound, but a star."

When the artist uses prose and poetry, imitating the material world in image and concept, he tries to convey to the spirit through intellectual processes a message that the wires will hardly carry. Fortunately the spirit pervades the whole being and it is this that makes it possible for language to express the ineffable. Sometimes the artist despairs, and resorts to symbolisms, hoping to make the symbol suggest instead of convey or imitate the vision, and in this procedure he is approaching the methods of mathematics. But whatever his material or his technic, the real artist always is creative and is endeavoring to show forth his vision, even if imperfectly. It is in some respects the glory of art that it suggests only, and does not convey the whole meaning.

Religious mysticism need only be mentioned. The works of the prophets of mankind are not unknown. We point out only that the essential character is not different in this case from that in the others.

The results of a sympathetic appreciation of the universe are not subject to what is usually meant by proof. We cannot prove that a sunset is beautiful, all that is possible is to say: "Look! Behold the floating golden cloud, and you will see it is beautiful!" Appreciation is direct, immediate, intuitive. We may illustrate the assertion by many cases from mathematics, for usually the so-called proof there will consist in merely pointing out simple relations one after another until the complex whole is seen to be true. If one cannot perceive the relations he will not understand the

"proof" at all. The more easily he perceives relations the less explanation is required, and a genius sees through the whole at one glance, needing no proof whatever. A long demonstration in mathematics, which involves a succession of steps so elaborate that it is not possible to perceive the whole unity at once is always lacking in real conviction. One will admit that there is proof, but the significance of the problem does not enter into the mathematical consciousness so as to become alive. To become a mystic of any type this power of direct insight must be developed. The ability to transcend the sense-data, to see through and beyond them, must be cultivated. He would be a poor physicist nowadays who could not look into the ultra-microscopic, and he is a poor astronomer whose vision does not reach as far as three hundred thousand light-years away. Relativity is mysticism of a scientific type, for it is the attempt to rid ourselves of the illusions of physiological and psychological space and time and to transcend the three-dimensional space wherein we eat, drink, and are merry, so as to appreciate the six-dimensional space in which we may float as an enormous ball. We may indeed assert that in the study of facts gathered in laboratories and observatories, we finally come to a point where the interpretation of them transcends all empirical laws, and we are forced to see the universe with that inner, mystic sight, which even the scientist possesses. In short, we either see reality mystically or we postulate (that is create) a reality which will be the substance of the phenomena observed.

This truth is evident to many scientists of the twentieth century. Consequently it becomes daily more apparent that Science is busied with only a partial study of the universe, and is compelled itself to depend ultimately upon the mystical perception, and that the great departments of human activity called, Mathematics, Anthropology, Art,

Religion, also have their place, and reach valuable conclusions. Even though their methodology is based largely upon the mystic insight, upon the spontaneous creativity of the human spirit, nevertheless they furnish a true view of Life and the Universe. Their mistake is to assume that they alone see the true, and the scientist's mistake is the same. Spiritual vision may be blind to material things and become ecstatic, visionary, and fanatical. Scientific vision may be blind to spiritual things, and become pragmatic, myopic, and bigoted.

Life is unitary, and not divided, and we need to take to heart the poet's message, a mystic's well-balanced view:
Let us not always say,

"Spite of this flesh today
I strove, made head, gained ground upon the whole!"
As the bird wings, and sings,
Let us cry: "All good things
Are ours, nor soul helps flesh more now, than flesh helps soul!"

Then, indeed, we can look at the floating feather-cloud and get a complete view of it, rejoicing in our understanding of the physics, chemistry, astronomy, meteorology involved, thrilling with our appreciation of the beauty of the scene, and able with the mystic sense to read the message it wrote:

Spirit is the only Reality.

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SYSTEMS OF AESTHETICS IN FRANCE[1]

"What is the beautiful? A blind man's question answered Aristotle." —*Emeric-David.*
Investigations into Statuary Art among the Ancients (1805).

IN EVERY domain of activity, for more than a century, our blunted human intellect seems to have lost all sense of actual reality and given itself up to memory and anticipation. It is creating a dull and turbid atmosphere wherein ideas have ceased to be the expression of life, and speculation no longer aims at the discipline of action. The parasitic development of reflection has brought about the appearance of conceptual systems. And just as metaphysics is ordering the world anew, so aesthetics purposes to set artistic activity in order, without even calling forth mistrust on the part of artists who are only too ready to become theorists, or of critics who regard their work as a sort of literary exercise. Here we have a strange fact, one that is peculiar to modern times and evidently but temporary, though it may be advisable to follow its birth, its progress and repercussions in French creation and reflection.

The humanist society, which began in France at the time of the Italian Renaissance in complete opposition to the Middle Ages and apart from all religious tradition, attaches too high a value to positive science to abandon itself thoughtlessly to the practice of the fine arts. No doubt Descartes reminds one of Père Mersenne, asking him for the reason of beauty, the relativity of aesthetic judgment,

[1] English translation by F. Rothwell.

and the variations of taste; he alleges that "as a rule neither the beautiful nor the agreeable mean anything more than a relation of our judgment to the object, and because human judgments are so different, it is impossible to say that the beautiful and the agreeable have any determinate or fixed value whatsoever." In the *Compendium Musicae,* however, he states that optics and acoustics enable the scientist to measure the relation between visual or auditory sensations and their object, to determine to what proportions that particular quality which makes sensations agreeable responds.

And so Descartes is indicating the path which artists should take, at the very time that the foundation of the Académie de Peinture in 1648 releases them from the rules and regulations imposed by the Corporation of Saint Luke and brings about a scission between the mechanical and the liberal arts. Painters and sculptors insist on being no longer confused with house-painters and stone-cutters, on being placed in the same category as writers on whom special prestige is conferred by the Académie française. As they are eager to prove the nobility and dignity of their task, they side with the taste for reason, scorn to waste time over technical matters and mean to go in for philosophical reflection. They approach the study of methods only in order to discern the part played by reason and judgment in the fine arts. The purely plastic comprehension of forms, lines and colors disappear in presence of the intellectual interpretation of ancient sculpture and architecture. And we have the passionate meditation of Poussin in the Roman Campagnia, the monthly meetings of the Académie when Le Brun comments before his pupils on Poussin's *La Manne* in the same way that some of our contemporaries comment on the work of Cézanne; we have the writings of Félibien. Monument, statue, picture are important only as expressing a spirit that recognizes the

invariable laws which the pressure of materials and the structure of the human body obey, building up by means of these intellectual data a whole whose parts are indebted to geometry, not to chance, for their proportions and their balance.

But this criticism, perfectly right in itself, claims to have a pedagogic value; analysis of the work would appear to give up the secret of creation. Between the understanding of art and the practice of it there has come about confusion which, ever since 1680, has resulted in reaction on the part of artists and writers on art, and in adhesion on the part of men of letters and "philosophers." The docility with which certain painters and sculptors adopt the thoughts of writers enables them to extend the war of words in which they are engaged, no longer to confine their reflections to the texts and to extend to artistic activity as a whole the conclusions of Cartesianism relieved by Fontanelle of its metaphysics. For the Cartesianism which gives them intellectual security seems to them a true and faithful expression of civilization. In the cult of the true, the conformity of man with his nature and his excellence, they have discovered a mental discipline, a rule of life. Indeed, admiration for antiquity gives umbrage to these systematists; does it not entail poetic fictions, a superstitious paganism, popular errors? And they pursue the deities with which Ronsard peopled the woods and Rubens made a procession for Marie de Médicis. They are resolved that the trees, the streams and rocks shall again become natural. They subject the models, so belauded by writers, to a minute criticism and allow them only one attenuating circumstance: that of having been in contact with nature. Hence they strengthen anything of a fluctuating nature there might have been in academic tendencies. The agreement between Perrault and Boileau puts an end to mutual misunderstandings and misinterpretations and brings

about unanimity of comprehension on questions dealing with the liberal arts, the great need of uniting artistic activity and scientific activity so that they may respond to conceptual requirements.

This attempt, however, to apply the philosophic spirit to the fine arts can develop but slowly and without a cause, as it were. In proportion as the fine arts see the birth of new tendencies, the greater becomes the need to resolve contradictions in taste. With the age of Boileau in the past, and the appearance of Le Sage and Rigaud, criticism, the warfare of words, and the quarrels of the last quarter-century lose the coherence which the classic ideal had given them. All that remains is a mass of contradictory tendencies, of useless rules which increase the fluctuations of taste, the eclecticism of amateurs and the hesitation of artists.

Then the philosophers offer to mediate. In pursuance of the reform brought about by Descartes in the intellectual realm, they enter into artistic activity in such a way as to give independent life to the conceptions which concentrate the cares and anxieties of the times.

In 1715, P. de Crousaz, a Swiss mathematician and philosopher who lived in Paris between 1682 and 1684, published a *Traité du Beau*. He finds fault with those who, having spent their whole life in gratifying sense and passion, gives the name of beautiful solely to that which charms their senses or fills their heart with pleasant emotions. These forget that art offers reason something which should please it and that the light of reason should have a share in our judgment. Now, temperament, amour-propre, habits, passions, our natural fickleness, the deterioration of the body and of the sense-organs are all incessantly opposed to the perfect concord which should prevail between senses, heart and reason. They cast us adrift, filling us with strange tastes from which we can protect

ourselves only by turning towards the light of reason. What reveals them to us is the practice of the physical sciences wherein the working of the logical organization enables us to unite in one system of truth a prodigious number of facts, it is an exercise of intellectual activity *which is not enclosed within the same limits as the senses,* even more precisely, it is the *language* of geometry which, alone, expresses real proportions, the development of mathematics "wherein the mind is delighted to discover uniformities which are held together by endless diversities." Indeed, the conditions of investigation, discovery and scientific certainty are to be found in the fine arts. Science and art employ one and the same logical organization. It is "the conjunction of several thoughts, their reference to one and the same end, their dependence on one and the same principle" that ensures both the *true* and the *beautiful.* Variety, unity, regularity, order, proportion, all geometrical and logical qualities, become of necessity the natural characteristics of the beautiful, since also, as de Crousaz writes, the conviction that the beautiful is essentially geometrical and that its origin must be sought in medication is based on conviction rather than on demonstration.

This transposition of Cartesianism, however, could not be the doing of a mathematician, and abbé Dubos thinks he acts "as a philosopher" by contenting himself with combining recollections of his travels, the precepts of Félibien and the reminiscences of Fontenelle in his *Réflexions critiques sur la poésie et la peinture* which appeared in 1719. In the Dutch school he points out the triviality of subjects as common as a basket of flowers, a man walking along the road, a woman carrying fruit to the market, a village festival, heroes who are merely scoundrels; he finds fault with pictures of the Lombard school because they do no more than flatter the eyes by the wealth and variety of their color; he notes imperfections in the Roman school, in the

work of Raphael and Veronese; finally he whimsically remarks that "Europe is nowadays only too full of silks and porcelains and other curiosities from China and Central Asia." Art is something quite different; it purposes to supply the craving for excitement and amusement within us with the food of superficial passions aroused by the imitation of human actions, by the expression of "faces that think in order to give food for thought," by nobility of subjects. The excellence of art in France for two generations, the constitution of European genius around France, testify to the fecundity and the durability of the classic doctrine common to painting and dramatic poetry. And so abbé Dubos simply gives fresh expression to the national claims of the "moderns" against ephemeral infatuation.

Reflection on the fine arts acquires genuine originality only in the case of abbé Batteux. Noticing how we are incommoded by a fund of observations and reflections on art, Batteux purposes to proceed like "true physicists who accumulate experiments and afterwards found on them a system which reduces them to a principle" in 1746, the very year when Condillac, in the *Essai sur l'origine des Connaissances humaines,* attempts to reduce to a single principle all that deals with the human understanding. Of course abbé Batteux was mainly occupied with linguistics, grammar, the poetic art and belles lettres. Doubtless he is aware, as was the time in which he lived, of the increasing vogue of music and dancing, as well as of the interests of the heart which "has a metaphysic of its own," "has its own intelligence which is independent of words." Still, in abandoning the remarks, reflections and speeches of a whole host of commentators for Aristotle's "Poetics," he at the same time frees himself from academic prejudices regarding the dignity of the fine arts and philosophical analyses bearing exclusively on the formal characteristics of the beautiful. Like Buffon, Rousseau and Lamarck, he

subordinates the partial point of view of knowledge to the more general point of view of action.

The nature of the arts, then, is explained by the genius of the man who produced them. All that is necessary is to go back to the start. To satisfy human needs, men have first been led to invent the mechanical arts. Then, to bring to an end joys of too uniform a type and to revive the stream of ideas, they invented the fine arts. But "to invent in the arts is by no means giving existence to an object, it is recognizing it where and as it is. And the men of genius who are most indefatigably searching discover only what existed previous to themselves. They are creative only in having observed, and reciprocally, they are observers only in order to be in a condition to create." The human mind, limited in fecundity and in its views, creates only in the wrong way and within the limits of nature: the mechanical arts are based on the use of nature, the fine arts on the imitation of nature and on the choice of its finest parts. Both presuppose a will, an intention, reasoned views, fixed rules confirmed by experience and suited to human nature. And so, while seeking reasons to admire in the *Beaux arts* and in *L'Art,* a dialogue addressed to Shaftesbury, abbé Batteux has discovered reasons to understand.

Thus, in the course of half a century, the blending of the philosophic mind with reflection on the fine arts first brings about a split between the liberal arts and the mechanical arts along with the sudden identification of artistic and scientific activity. Thorough investigation into their nature reveals progressively that, while the fine arts, like science, call for meditation and intellect, they none the less retain purely technical aspects which relate them to the mechanical arts. And, about the middle of the century, the fine arts win their autonomy in the hierarchy of the sciences

and arts, diverse expressions of human activity which is fundamentally a unity.

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Nevertheless, affirmation of the intellect has been too complete not to carry along with it a mysticism of the intellect. On the other hand, while the influence of Descartes was strong enough to impose a discipline on the religious enthusiasm of Malebranche, none the less did it leave existing a mysticism inspired by Plato, Plotinus and Saint Augustine, revealing the dependence of the human mind on God. This current acquires a fresh power of suggestion when the time comes for the effort of innovation made by artists and amateurs to remove all trust in academic doctrine and give ear to sensibility. The ideas devoted to the powers of feeling are again imbued with desires and melt away. Quite naturally does Père André, a Jesuit seduced by his partiality for the Oratoire and the *Recherche de la Vérité*, give an anti-intellectualistic orientation to reflection on the fine arts in his *Essai sur le Beau*, which appeared in 1741. Conscious of the transformations that have happened in art, of intellectual distress and wavering minds, Père André employs various amplifications and paraphrases in attributing to beauty the doctrine which Malebherbe offers of vision in God, within the physical and the moral order of things. As each truth dwells in God and each individual reason is but a reflection of universal reason, beauty, the idea of which is found in all minds, could not have a human origin. The beautiful as man arbitrarily accepts it, and the beautiful as manifested in the great field of nature, are but reflections of an essential beauty, independent of all constitution, even though divine. It is an eternal pattern or model of the visible beauty of bodies; and the love of the beautiful, which appears along with reason, is explained by the theory of

reminiscence offered by Plato, who "as the Fathers of the Church have remarked, had read the books of the Hebrews, especially Moses and Solomon." It is the Creator who imprints on all souls the love of the beautiful, and a special sort of love for a certain kind of beauty. And so, contrary to the spirit of Cartesianism, reflection parts company with the positive sciences in order to rely on revelation, religious meditation, mystic experience, looking for the essence of beauty in a principle of perfection and excellence, in something which is independent of and superior to man, something which is divine.

A parallel stream of thought may be found in the Anglo-Saxon peoples. The influence made upon them by the Renaissance has not been sufficiently direct to break all links with the Middle Ages and their scholastic habits of mind; the Reform made them partially alien to catholic discipline. They have witnessed, even contributed to the development of the positive sciences without seeing in the spirit of mathematical physics the elements of a mental discipline capable of restraining the outbursts of the imagination and of imposing a rational order on human activity. They have witnessed the discovery of Antiquity without understanding the lesson of balance therein revealed. In the course of travels in Italy, their countrymen, incapable of solving the problems of Venice and Florence, find in Rome nothing but ruins calculated to favor meditation on the Empires, to appeal to archeological investigations, to indulge in sensual day-dreams. For the representation of the human body seems to them devoid of intellectual and human value. It is an animal appeal, from which nothing but a religious, moral and metaphysical interpretation of plastic art can divert and turn us aside. And it is the Platonician and Neo-Platonician reminiscences, combined with religious feeling, that enable such Englishmen as Shaftesbury and Hutcheson, such Germans as Winkel-

mann, Raphaël Mengs and Lessing, to restore all its purity to their irrational enthusiasm for the antique. Then only does the supreme beauty which we thought we had apprehended in particular forms and in the appearances of sense, reveal itself as inapprehensible, ineffable and divine.

This stream enters France owing to an active exchange of ideas, and more especially to Diderot. No doubt this unsettled, fickle and impressionable mind again makes his own certain precepts of classic art, whose vitality was assured in the work of Gabriel and even under the Empire in the works of the Ideologists. But he is sensitive, he is moral. It fills him with indignation to see the painter Lagrenée adjust Truth, Virtue and Religion to the boudoir of a financier, and to find Boucher offering for public favor "his indecent and vapid marionnettes." There we have a decadence of art due to the excessive confidence of painters and amateurs in experiment and study. It is the part of the sensitive refined man, who knows that "the true, the beautiful and the good practically have their roots in one another," to bring about a reform of art by a return to metaphysics, an appeal to the ideal. *Les Recherches philosophiques sur l'Origine et la Nature du Beau,* and afterwards *Les Salons* inaugurate a sort of literary dissertation on art offering an ensemble of more or less incoherent suggestions, though their effective share in this development of intellectual dissolution and desocialization which we somewhat erroneously call romanticism was assuredly considerable.

The effervescence of Roman environment, the works of de Chastellux, and social events all hasten on this reform which seeks in antiquity for grounds of exultation and claims to have restored heroism. Artists and amateurs handle bas-reliefs, medals, cameos and painted vases, which some irreverently call Etruscan pitchers, with antiquarian minuteness and a solicitude for exactness that paralyze all

play of imagination. They ask for inspiration from a spiritualistic metaphysics, a naive morality. The return to the antique extolled by Vien is a sign of incapacity to liberate speculation from literary and moral commentaries and to recognize the preoccupations of a Poussin. Powerless to control the fluctuations of taste and the contradictions of his nature, David enters from without upon Greek life enshrined in stone only in order to combine republican austerity with the blandishments of Coypel. And French art is confined to the imitation of forms at a time when the spirit of the Renaissance is already becoming too remote for any understanding of Greek statuary to be possible.

The effort of thought and of art to return to nature through a new image of the ancient world is prolonged in spite of the protests of Emeric-David, who reacts against the exaggeration and emphasis peculiar to the theorists of ideal beauty, by seeking, in the history of Greek society, the causes of its plastic perfection, and regarding its humanity as the secret reason of its attractiveness. Very learnedly in 1805 does Quatremère de Quincy refute his theory in the *Essai sur l'Ideal* by showing that imitation ought to be *ideal* in the modern world as it was in the Greek world, where, he says, "religion was the principle motive and the propagator of the style of ideal imitation"; he supplies artists with advice on the way to generalize, to metaphorize, to realize the ideal plan, to conceive the ideal costume,—advice which is but too scrupulously followed out. And before long, Madame de Staël, dealing with all countries and all epochs of history, North and South, Antiquity and the Gothic Middle Ages, is to extend the bounds of human destiny and, following on Kant, bring into the fine arts a sense of the infinite.

From 1815 onwards, Cousin turns all these streams into one by combining the suggestions of Père André, Winkelmann, Diderot and Quatremère de Quincy with the theo-

ries of Schelling and Hegel. He is not content to set up before the individual principle of imitation the absolute and general principle of creation; he identifies art with the search after spiritual beauty of which nature is the *symbol.* There is a God within us; there is a God without us. It is for us to raise the veil, to understand the symbol in a burst of intelligence and love wherein artistic activity, metaphysical meditation and inspiration are blended into one. On this basis, Cousin elaborates a wholly aesthetic doctrine whose spirit is to survive in a century unskilled to understand the great lesson of Ingrès (whom we are only now beginning to reveal) when he abandons physics for chemistry and biology which give science a new direction, one that is rather empirical, than experimental.

Art as the expression of ideal beauty becomes a privileged mode of knowledge, more direct and immediate than scientific knowledge. Intuition takes the place of deduction; the regularity of logical proceedings is broken. Successive bounds and nervous jerks supply the elements of an artistic symbolism in which the broken hierarchy of concepts hands over the world and mankind to the impassioned interplay of individual forces, spontaneous and wholly undetermined. Now nature becomes humanized; now humanity melts away into the one universal life. All the time, however, secret correspondences and analogies are set up, reviving the mysticism of the Middle Ages. This mysticism permeates the work of Jouffroy, Taine, Sully-Prudhomme, Guyau, and the transposition thereof in the German theories of the Einfühlung and the Expressionism of Benedetto Croce. In all alike, art is still an expression of deity and the soul.

True, from 1860 onwards, the new favor enjoyed by physical science, the evolution of art, the influence of Darwinism and the New Criticism seem to restore a positive spirit to reflection on the fine arts. There comes into being

a mighty development inspired by studies originating in the *Aesthetica* of Baumgarten, and principally in the *Critique du Jugement*. Combining philosophical speculation with scientific observation, it attempts to determine the psychological and physiological conditions of the judgment pronounced on the beautiful. Along with Kant, this development brings into the same category artistic activity and the free disinterested play of mental images which combine, blend and separate, in accordance with the developments of coenaesthesia, the course of day-dreaming, the will of caprice. Reduced to idea alone, it becomes nothing but a formal mechanism without any definite links with biological life, the outer world and human environment. Because it has failed to apprehend artistic activity in its own distinctive nature, it has had no repercussion on this activity. As such, it is one of the signs of increasing desocialization from which we have had to suffer throughout the whole of the nineteenth century; it brings out the divorce that has come about between the various modes of human activity, the barrenness of doctrines which substitute the development of a dialectic for the mutual influence or action of thought and society.

In spite of its intentions, then, *the science of the beautiful* unwittingly opens up a wider field for the *metaphysics of the beautiful*. Both alike appear during the eighteenth century. They have their roots in the imaginary recreation of the interior life which science remains powerless to attain, which thought endeavors to penetrate in order to cheat its craving for the romantic. Gradually and right up to the present time, they discredit reflection on the fine arts which springs from Cartesianism. In the confusion of ideas and the complexity of tendencies they find reasons for continuing to exist. In giving themselves up, however, to the emotions roused by a work of art, and in allowing contemplation free play, they confine themselves within a

mode of abstraction unrelated to artistic activity; they condemn themselves to remain wholly out of touch with Humanity.

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The fact is that, ever since the fifteenth century, the crisis traversed by Greek thought in the days of Socrates has been repeated in the modern world. The acquisitions of the physicists and the moralists, the sophists of the present times, are simultaneously extolled and questioned by two civilizations which defy one another and contend for the control or guidance of the human mind. There is the Renaissance, which regards the fullness of human effort as consisting in a fusion of the arts, the fine arts and the sciences, and which looks upon the harmonious balance of intellect and life as depending on accepted disciplines. And there is the Reform, which considers that our wisdom and our destiny depend upon the spontaneous expansion of consciousness. Hence the oscillations of Western Europe, wavering between intelligence and sensibility, between the positive sciences and metaphysics. Hence, the intentions which press heavily upon an art that is too overburdened with ideas, beliefs and passions ever to regain the sobriety of Grecian statuary, the plastic purity of a Veronese, the serenity of Johann Sebastian Bach. Hence the gropings and wanderings of artists in whom professional scruples, reflection and conscience have sunk so low as to allow of improvisation; the wanderings of philosophers who have become so forgetful of the limits of the mind and of the blindness of the senses as to become indebted to coenaesthesia or to imagination for their escape from the human. On three occasions, about the years 1750, 1810 and 1860, the coalition of the powers of feeling that came from France, England and Germany, sets up against Cartesianism the spirit of Kantism, throws back artistic activity on to this or the other side of life and inflicts a

check on humanism. And on three occasions humanism revives.

Latterly, the orientation of art seeking to permeate modern life, its scientific spirit, its machines, its ever unstable social groupings, the restlessness of Cézanne, Rodin and Debussy have restored artists to a liking for mediation. The combinations of lines, colors or sounds, the balance of volumes, become the object of reflection which is at times sufficiently exclusive to function *outside the limits of mental images.* Here, between the technical means of art and the human conditions of artistic creation there is a confusion reminiscent perhaps of the deliberate purposes of Le Brun and the *Académie,* though attesting to the will to restore to artistic activity, corrupted and debased by the aestheticians, its original purity. Is this will sufficient? Does not any restoration of the plastic arts take for granted what was lacking yesterday and what is also lacking today: an expansion of human environments that is, an intellectual and moral development, sufficiently intense to snatch the individual from organic rhythms, from the life of imagination, from the purely formal mechanisms of thought, and to feed his passions.

RAYMOND LENOIR.

CREATIVE MONISM

NATURE as empirically presented to sense awareness is given in the form of qualitatively different kinds of sense data. It is only by intellectual activity that we discover and substitute for this "sensible muchness" the unifying underpinning of qualitatively uniform elements of nature, which are then taken to be the ultimate "stuff" of reality. The view that the various kinds of reality are not reducible to one ultimate reality is, of course, embodied in various forms of dualisms and triadisms, but it would probably be admitted by all that monism is to be preferred to any other view, provided it can do justice to all the facts. Insofar, then, as this stuff of reality is one kind of substance it may be viewed as either physical or psychical "stuff." However, neither physical nor psychical monism have succeeded in irrefutably establishing their positions. The rock upon which psychical monism founders is encountered in the attempt to explain why the laws of what we call "matter" should be so different from the psychical, and why the so-called material organization of the universe should be historically prior to the mind which appears as a relatively late product in the evolutionary scheme. On the other hand, the nemesis of physical monism is the inability to incorporate within the system of the homogeneous units of physical science this same intellectual activity, which seems so different from the universe of matter it seeks to interpret. Neutral monism takes its rise as a

proffered solution to the difficulties of physical and psychical monism and the riddles created by dualism. By forcing a wedge of neutral stuff between mind and matter, which stuff, by definition, is endowed with all the unit characters necessary to account for the innate characteristics which both mind and matter display, the neutral monist then steps across the chasm, straddling both the psychical and the physical. How this hybrid stuff becomes fecund the neutral monist does not tell us. As for the identity theory, critics usually dispose of this view with the statement that it is explaining the relatively known by the absolutely unknown.

To establish a monism which obviates the difficulties of the various forms of monism it is necessary to provide a place for the appearance of new qualities in the course of evolution. A view which provides a place for the appearance of creative novelties in the historical development of nature satisfies an ethical as well as a logical demand. The reason naive materialism is rejected is not out of deference to current ethics, but because it is not in accord with more enlightened views of objective nature. Nevertheless, to minimize the prejudice against the view which I have termed *creative monism*, it is necessary to distinguish it from the billiard ball conception of ancient atomism and its modern descendant mechanistic materialism. In redefining the concept of physical nature I am following more or less closely the views of Doctors Whitehead, Broad, and Alexander. The view of concrete nature as a progressive movement towards the production of "higher" complexes with "new" properties or "emergents" has more recently been developed in C. Lloyd Morgan's theory of *Emergent Evolution*. The method adopted in considering the view of creative monism will be (1) to state the dubious position which qualities occupy in contemporary thought; (2) to consider the historical development of the subject of pri-

mary and secondary qualities; (3) to state the "method of extensive abstraction" as it bears on our problem; (4) to expound the view of nature as a system of durations; (5) to portray the positive picture of the world as it exhibits itself in the view of creative monism.

I. The Status of Qualities in Modern Thought

If we could personify quality, we might call it the Wandering Jew of philosophy—homeless and with no place to lay its head. The physicist has eliminated colors, odors, sounds, tastes and temperatures from his world of positive and negative charges of electricity. His is an abstract world of quantities denuded of all the wealth and richness that we think we perceive in nature. Quite naturally, we suggest that if qualities are not in the external world they must be in the perceiving organism. Therefore, in what some would call traditional psychology, room was made for the sensed qualities, which the physicist expunged, in the concept of sensations as elements of conscious states. Now, however, behavioristic psychology, rejecting the notion of mind as constituted of elements with qualitative attributes, substitutes sensory discriminations of various wave-lengths of light for "colors," and analogous explanations for other "experiences" of sensory content. At the same time the behaviorists have refused to enrich the environment with the cast-off garments of mental content. But however you explain colors, all must admit that these terms refer to some sort of differential activities or responses which go on in an organism when it perceives or reacts to qualities. That seeing red is nothing but the muscular response we make in reacting to light waves which set up a specific photo-chemical or photo-electric activity in the visual receptors critics of behaviorism can

not admit. Similarly, thinking may be a form of "behavior," but it is precisely because of the inability of the radical behaviorist to find qualitative differentia for the contents of "mind" that the present writer rejects behaviorism as it has been expounded up to the present.

We speak of objects as being red or green, bright or dull, hot or cold, sweet or sour, etc. Where, then, are these qualities which, to appear, demand both an organism and an environment? Is the green of the leaf an attribute of the leaf? The answer to this question is not as obvious as it may seem. The naive realist believes that the green is really in the leaf. But the physicist asserts that color depends upon the light reflected from the leaf, and this reflected light we call green. The physiologist knows that the leaf which appears green when looked at directly may appear yellow or gray when the image falls upon the peripheral part of the retina. He is therefore inclined to regard color as an attribute of the eye. Finally, to the structural psychologist, the green is neither an attribute of the leaf, nor the light, nor the eye, but a definite qualitative entity in consciousness.[1]

Now if the physical monists in psychology insist upon pushing qualities, as such, out of the experiencing organism, and if the physicist will not tolerate them in his universe, where in the world are qualities? Or are qualities, like the fictitious creature of mythology, only imaginary entities kept alive by philosophers to have something to dispute about?

II. The History of Primary and Secondary Qualities

One of the most obvious things about the universe is that it is constantly suffering change, but that in the midst

[1] *Introduction to the Study of Color Vision*, by J. H. Parsons, New York, 1915, p. 22.

of change there are foci of permanence. To explain this problem of change it has been the natural tendency to postulate some underlying substratum as the seat of qualitative changes, which are then regarded as transformations of this primal stuff. One of the earliest problems of Greek philosophy was to describe the nature of this original essence. The categories of substance and quality first appear explicitly in Aristotle's system, who, therefore, is credited with fixing in human thought the idea of the *thing* as the bearer of the qualities which inhere in this substantial substratum. While this is not the place to consider whether this is a fair interpretation of Aristotle we may note that the new realists, on the whole, argue that the notion of matter as the metaphysical substratum of its properties,[2] or of substance as a thing-like core in which qualities inhere,[3] is the consequence of the traditional Aristotelian logic of classes.[4]

This substance-quality view has affected all subsequent philosophy and science. One needs only to note that it is the metaphysical basis of the religious doctrine of transubstantiation to indicate its importance in Western thought. Perhaps, also, the contempt for matter as a principle of evil is to be sought in the turn which the Greeks gave to the problem of *being* and *becoming*. Both in Plato and in Aristotle a dualism appears between the purposive activity of the *idea* or *form* and the resistance of matter. In science this notion of matter as a retarding principle is found in the concept of inertia. This stands out in the classical mechanics in which the substantial substratum of inertial mass is considered as coinciding with the primary qualities of spatial extensity, impenetrability, etc. The substantial-

[2] *The Concept of Nature*, by A. N. Whitehead; ch. I.

[3] *The New Rationalism*, by E. G. Spaulding; pp. 29-35.

[4] In a proposition, which is the functional unit of the Aristotelian syllogism, the subject and predicate terms are class names, and in a logic of classes the predicate is the affirmation or denial of a quality or attribute to the subject term.

istic view has exercised its preëminence in determining the view of the nature of space as the vessel or container in which the motions of "matter" occur; of time as the history of the transformations of matter in space; of force as the active cause of the motions of matter; and of the aether as the underlying continuum of the interactions of the bodies of nature. It is only recently that we have sufficiently disengaged ourselves from this attitude to permit ourselves to ask whether the notion of a thing-like stuff is the ultimate concept or whether events, relations or qualities are the final ontological realities. Without for the present following out the modern mechanistic view of the external world, finding its exemplary expression in the systems of Newton and Laplace, which grew out of the notion of substance as the seat of primary qualities, we will return to our original purpose of tracing out the development of the dualism which makes secondary qualities subjective and primary qualities objective.

From the time of Democritus, who first assigned to secondary qualities a subjective status, primary qualities have been taken to refer to those properties of things which enable the physicist to manipulate mathematically the more permanent aspects of the external world. Thus the physicist abstracts the permanences from the transformations of nature and summarizes these processes in statements of quantitative equivalences. The mechanical theory rests upon an analogy between the qualitative changes of experience and the movements of material particles in space. In the early period of modern thought the subjectivity of sensory qualities is recognized by Galilei, and is embodied in the views of Descartes and Hobbes. The effect of Locke's view has been to reinforce this distinction. Berkeley next proceeded to show that secondary qualities are on the same level as the primary qualities and that if we make the secondary qualities mind-dependant, then the so-called pri-

mary qualities are just as mental. In contrast with Berkley's epistemological monism the representative theory of perception affirms this dualism of primary and secondary qualities. Later on, in Kant, the idea of matter as the substantial source of sense-qualities is the basis of the thing-in-itself which has lost even the qualities called primary by Locke, and remains unknowable in character. However, Kant formulated the functional view, on the phenomenal side, in his definition of the *thing* as the law of its states. Hegel carries this view to its full consequences in denying entirely the existence of the thing-in-itself, holding that the thing is nothing but the systematic totality of its qualities.

In the field of psychology the notion of a sensation as an element of consciousness having certain attributes, such as quality, intensity and duration, and as bearing a proportional relation to the magnitudes of the stimulus, was definitely formulated by E. H. Weber. Weber's law states that there is a constant ratio between the scale of sensible intensities and the increase of intensity of the physical stimulus. Fechner continued the work of Weber and by assuming that the "just noticeable differences" in the scale of sensible intensities were equal Fechner gave Weber's law the well-known mathematical form, the sensation varies as the logarithm of the stimulus. The notion of the qualities of sensation as the subjective correlates of the magnitudes of the primary physical system was supplemented by Johannes Müller's doctrine of the specific energies of the nerves. This doctrine states that the quality which characterizes any mode of sensation is dependent upon a specific *quale* of the nervous energy, and not upon the method of stimulation. For example, mechanical stimulation of a sense organ (as the eye) may yield the same (visual) sensation as will chemical or electrical stimulation of the same receptor. Wundt also gave psycho-physics and the doctrine

of sensations a powerful impetus. The historical denouement of this type of psychological theory appears in the sensationalistic phenomenalism of Karl Pearson and Ernst Mach, which, in turn, is the progenitor of "neutral monism."

The doctrine of the specific energies of the nervous system does not occupy as important a place in contemporary psychology as it formerly did. Why this is so will appear in the remainder of the present discussion, which is largely a summary and criticism of the matter as set forth in E. B. Holt's[5] essay in *The New Realism.* Holt's article presents one of the most persistent attempts to reduce qualitative differences to quantitative magnitudes which has yet been undertaken.

The idea of nerve energies qualitatively differing for each of the senses must be rejected because the theory meets with great difficulties, especially in visual and auditory experiences, and because nerve physiology has been unable to discover any qualitative differences among nerve impulses or suitable chemical or histological differences among cortical cells or synapses. On the other hand, recent discoveries indicate that the nervous impulse, particularly the sensory impulse, presents fluctuations of a high frequency. Holt would substitute the *neural periodicities* or *physiological densities* theory for the specific energies theory. Holt admits that there are qualitative differences, but holds that these qualitative wholes can usually be analyzed into quantitative differences. The doctrine against which Holt directs his attack is the doctrine of *Gestaltsqualitäten* (*form-qualities*), a view which has affinities with the theory of *creative synthesis* and Wundt's doctrine of *psychical resultants.* In the present connection the doctrine of form qualities refers to that which characterizes a mental whole or higher psychic synthesis as having a unique or unanalyz-

[5] "The Place of Illusory Experience in a Realistic World," by E. B. Holt in *The New Realism*, pp. 303-373.

able factor present which is not to be found in the separate parts.[6]

In showing how the "uniqueness" of qualities organized in space and time is nothing but this *organization,* Holt[7] uses for illustrative purposes a series of touch stimuli given at an ever-increasing rate. A single tap possesses a quality. A pair in close succession is already a form-quality, which differs from the first by just as much as the difference in temporal organization has altered it. As the rate of tapping increases the unified aspect approaches the quality of roughness, and the succession of parts aspect becomes vaguer. Thus, in the experience of tactual roughness the properties of the ordered whole gradually supersede in attention the properties of the parts ordered, not because the whole, as the rate of tapping increases, is coming to be anything other than the sum of its parts, but because the quicker succession soon eludes the sense of time, and so leaves other features of the succession to occupy the attention. The rise of form-qualities can also be found in auditory experiences as a succession of sounds passes into a tone with an increase of frequency. Attentive introspection, insofar as it relies on the time-sense, has its distinct limitations. According to Holt, secondary qualities are not something over and above the stimuli as given in a time sequence ("neural periodicity") but "it is some intrinsic feature of that succession, logically bound up in it." When the time-sense or attention can no longer discriminate the successive aspect the experience is given a totality aspect, perhaps by mere summation. This implies a completely atomistic psychology of nervous shocks as Holt, following Spencer, admits. Before proceeding to criticize Holt's theory of qualities it is advisable to observe how this

[6] For *Gestaltsqualitat,* the term originally suggested by Ehrenfels, Meinong uses the term "funded content" (*fundierte Inhalt*). See Baldwin's *Dictionary of Philosophy.*

[7] *Op. cit.,* p. 340.

doctrine squares with his general position established in his *The Concept of Consciousness.*

The neutral monist starts with a universe as an eternally subsistent realm of mathematical and logical terms and relations. In order to make the transition from this static realm of neutral elements to the concrete world of evolution, Holt endows these logical propositions with logical activity. The more empirical complexes are then generated out of the abstract non-mental logical entities. The passage from the simpler to the more complex empirical systems in space and time constitutes an asymmetrical relation. For example, carbon, hydrogen and oxygen can exist without being starch, but starch can not exist without being these three elements. Higher complexes are nothing but the constituent elements plus their organization. The universe is a hierarchy of ordered series of graded complexities. For neutral monism[8] the secondary qualities, forming linear serial orders of colors, sounds, odors and tastes, enjoy the same ontological status as the primary qualities. So far, Holt is consistent with his view of qualities as wholes which get their character from the ordering principle which generates them.

The main criticism which is to be made of Holt's position (aside from the fact that he has not told us what the "time-sense" is) is that he has not clarified the relation between logical activity and the temporal processes of nature. The difficulty which all neutral monists face is that of explaining how logical entities can be eternally subsistent and still generative of empirical reality. The difficulty inheres in the very assumption with which they start—it is impossible to build a universe out of neutral stuff. I shall argue later on that the ordering relation which constitutes the uniqueness of synthetic whole (qualities-in-relation) *is* the temporal-creative process of nature.

[8] *The Concept of Consciousness*, p. 134.

III. The Method of Extensive Abstraction

The view of external nature which we are here setting forth is that which Doctor Whitehead has presented in his three volumes, *The Principles of Natural Knowledge, The Concept of Nature,* and *The Principle of Relativity.* Without attempting to expound the mathematical justifications for the view of nature as the passage of events, we will consider principally those aspects of his doctrine of the creative advance of nature which bear upon the theory of creative monism. Doctor Whitehead's theory has been restated and expanded in C. D. Broad's exposition of the "method of extensive abstraction" in his *Scientific Thought.* There are no obvious departures in Broad's work, and I shall pass from Whitehead's to Broad's treatment whenever clearness of exposition is facilitated. The difficulty which attends the attempt to understand this important contribution to the philosophy of nature is that it is not easy to attain the detachment from traditional and uncriticized notions, especially those associated with the materialistic theory of nature, which is demanded by Whitehead's exposition. Before proceeding to this novel point of view a few remarks concerning the value of this outlook will be pertinent in serving to furnish us with a plan of development in our exposition. The advantages which the method of extensive abstraction possesses may be stated as follows:

(1) It promises to settle once for all the controversy concerning the relation of mathematical manifolds to the manifolds of the physical universe. This method is definitely superior to Bertrand Russell's solution of the problem of infinity and continuity in this respect that it presents a theory of the relation of mathematical continua to the spatial and temporal continua of nature which seems

acceptable to both realists and idealists. Russell is always hesitant about stating unequivocally whether physical space and time are constituted of mathematical points and instants, or whether points and instants are merely logical fictions.

(2) It starts with nature as it is concretely given in perception. This view makes no claim upon our imagination by seeking to conjure up sense-data out of abstract, non-perceivable entities—whether *world-lines*, *space-time*, or *neutral stuff*—which no one except the absolute could experience.

(3) It provides an adequate starting point for the theory of the creative advance of nature as the emergence of higher complexes out of the "cruder" materials of nature. Those who believe in "creative synthesis" as the explanation of the appearance of wholes which have a unique, unanalyzable quality, and which is not the resultant of mere summation, have frequently been content to state this synthesis as a bald fact which admits of no explanation. In a sense, the creative advance of nature is inexplicable; but what we have to insist upon is that this synthetic production of novelties is not confined to any restricted region of operation, like the mind, but is a pervasive feature of nature throughout.

With these preliminary matters disposed of, we may now turn to the presentation of the method itself.

The remarkable thing about the evolution of science is that the laws of geometry and mechanics come out of the study of the external world, and return to it in the form of applied mathematics. To explain this relationship, Whitehead and Broad present the principle of extensive abstraction as the method by which the accurate and refined concepts of science and mathematical physics are defined in terms of the crude facts of sense, or perceptible objects and their perceptible relations.

So long as the crude data of sense continue in the unanalyzed state it is impossible to reason about them. In order to organize these facts into a system it is necessary to analyze finite figures and their complicated perceptible relations into sets of terms with simpler and more manageable relations. A simple principle gives us the clue to the proper method. We find that space, time and matter become more manageable as we make the objects smaller and the relations between objects simpler. The goal of such a procedure would be parts with no size and events of no duration. As Whitehead[9] puts it, the method of extensive abstraction in its sphere achieves the same object as does differential calculus in the region of numerical calculation, namely, it converts a process of approximation into an instrument of exact thought. The procedure is to formulate the law by which the approximation to events sufficiently restricted is achieved and indefinitely continued. The complete series is then defined and we have a route of approximation. It follows[10] from the principle of convergence to simplicity with diminution of extent that, for exhibiting the relations between events in their utmost simplicity, abstractive elements of minimum complexity are required, that is, elements which converge towards the ideal of an atomic event. An event-particle is the route of approximation to an atomic event. It is the linear chain of transition through the continuum of nature. The difficulty of explaining how finite volumes can be made up of infinitesimals of no dimensions is avoided by pointing out that the usefulness of points, which, in themselves, may possess a complex internal logical structure, depends, as Broad indicates, entirely on the fact that any pair of them define a unique relation with very simple logical properties, viz., the straight line joining them. That is, the conditions laid down by the method of extensive abstraction for the definition of points,

[9] *The Principles of Natural Knowledge*, p. 76.
[10] *Ibid.*, p., 121.

lines, surfaces and volumes is, first, that they must have to each other the relations which these things have in geometry, and secondly, it must be possible to give a clear meaning to the statement that finite volumes and areas can be completely analyzed into sets of points. The general principle, in application, is the same. Points, straight lines, and areas are all defined as series of converging volumes. But series that define points differ in certain assignable ways from those that define straight lines, and these, in turn, differ in certain assignable ways from those which define areas.[11]

Ordinary perception gives us series of volumes which we must analyze into formal logical terms, and we do this by grouping into classes the respective volumes which contain each other. Through each series of containing volumes, whether they be cubes, circles or spheres, it is possible to find certain relations which hold between the volumes of the series. As these containing volumes grow smaller and smaller they converge to the particular type of relation which defines this volume. Some series of volumes converge to points, some to lines and some to areas. A point, then, may be defined as the class of all volumes in any series that would commonly be said to converge to that point. It is true that the points can not be perceived by the senses, but this does not make them unreal. Points exist in the sense that they are determinate functions of real series of existing particulars. The various other objections which might be raised against this procedure—such as that these definitions are circular, etc.—are anticipated and dealt with by Broad. So much, then, for the method of extensive abstraction.

[11] *Scientific Thought*, by C. D. Broad, p. 45.

IV. Nature as Duration

As Doctor Whitehead says,[12] no one can study the evidence in its detail without becoming convinced that we are in the presence of one of the most profound reorganizations of scientific and philosophic thought. It may equally well be said that no one is contributing more to this reorganization than Doctor Whitehead himself, who is doing much to precipitate out the muddy elements from the stream of traditional scientific thought. The only justification for the present attempt to sketch Whitehead's view lies in the desire to carry the sweep of this outlook into a broader field wherein investigators of a more empirical trend are too cautious to venture. However, it is always the privelege of the "synthesizer" of the sciences to rush in where less ambitious scientists fear to tread.

The ultimate fact of experience is a complex of passing events, Whitehead holds. What we are immediately aware of is a duration of nature with temporal succession. "The fundamental assumption to be elaborated in the course of this inquiry is that the ultimate facts of nature, in terms of which all physical and biological explanations must be expressed, are events connected by their spatio-temporal relations, and that these relations are in the main reducible to the property of events that they can contain other events which are parts of them." [13]

But the fact that a passage of nature is present to sense awareness does not carry with it any doctrine of the disjunction of nature and mind. Doctor Whitehead protests vigorously against the traditional "bifurcation" of nature into two systems of reality, that which is given as external cause, and some psychic addition furnished by mind. Per-

[12] *The Principle of Relativity*, p. 67.
[13] *The Principles of Natural Knowledge*, p. 4.

ceptual knowledge is nothing else than an apprehension of the external relatedness of things.

The most interesting and puzzling aspect of this theory of nature as the passage of events is the theory of material objects. Events are the ultimate substance of nature, and an apparent material particle, we are told, is an adjective of appearance which qualifies events.[14] Time, space and material are adjuncts of events; time and space spring from the relation of extension, and express relations between events. This does away with the need for the aether of space. The material called aether is merely the outcome of a metaphysical craving created by the materialistic theory. The *aether of events* may be substituted for the *aether of stuff* to express the fact that something is going on everywhere always. *Events* are analogous to Alexander's empirical groupings of Space-Time, except that for Whitehead "matter" seems already to be implicated in events, and not an emergent which appears as a complexification of an historically prior stuff, Space-Time.

The physics which results from this type of metaphysics would be termed the physics of the field. This kind of dynamics, which is assimilated in various interpretations of the principle of relativity, finds its origin in Faraday's conception of electro-magnetic tubes of force interpreted as streaming through space and time. On the assumption that the field of the electron extends throughout space the theory of action at a distance and action by transmission through a medium are reconciled. Just as the field extends through space so it also streams through time. The physical field is the distribution of nature at a moment as it expresses the character of the past reaching into the future. The field sustains the relatedness of things and constitutes the uniformity of nature. Spread through a spatio-temporal region, it modifies the ingression of objects. A physi-

[14] *The Principle of Relativity*, p. 69.

cal object, such as a mass particle or electron, expresses the character of the future so far as the future is determined by the happenings of the present.

Nature is the progressive advance in which higher objects emerge in their respective time systems. Nature is the concrete system of durations which transcends the particular time systems into which it may be analyzed. In this respect, Whitehead is in accord with Bergson's doctrine of duration. Whitehead distinguishes between the creative advance or passage of nature and any special time systems. The group of all time systems embodies the physical properties of this creative advance.

Since nature is unified by the pervasive fields, which spread through the adjacent spatio-temporal regions, it would be a mistake to construe the slabs of nature which are given in awareness as the logical sums of the abstractive factors into which they are analyzed.[15] The atomic entities of analysis are the logical ideals which are deduced as limiting notions. This limitation within the totality of the continuum of events is possible because nature has systematic structure. Here Whitehead's doctrine approaches the idealistic view of the internality of relations. He holds, however, that we can express the truth about the individual facts of nature without knowing the totality of events to which each item is related by distinguishing between the contingent and the essential relationships of the factors given in awareness. This is similar to Bosanquet's notion of relevance.

The criticisms which might be urged against Doctor Whitehead's view would rest mainly upon what Whitehead has left unsaid, rather than upon what he has committed himself to. Perhaps it is because he has deliberately excluded the problem of the relation of the mind or the "percipient event" to the events of the external world, with

[15] *Ibid.*, p. 15.

which the percipient event is "cogredient," that Whitehead's theory seems quite invulnerable. But even as it is, the theory of objects is tantalizing to the reader. Objects are ingredients in events which do not pass. But pass for whom or what? and why don't they pass? The theory of relativity suggests that the permanence of an object is due to the fact that it shares in the passage of a time system which is moving at the speed comparable to the speed of the observer's frame of reference. What may be an object for one time-system may not be an object for another time-system. Russell[16] admits this in the following words: "We can not speak in any accurate sense of the 'history' of a piece of matter, because the time-order of events is to a certain extent arbitrary and dependent upon the reference body." Moreover, Whitehead gives us reason to suppose that "mind" does not play an entirely passive role in perception, for, as he says, it is through the breakdown of the continuum of events into atomic entities that the mind isolates its problems. Whether the mind imposes this atomicity upon the otherwise continuous events (as Bergson holds that the geometrizing intellect only rediscovers the discreteness in nature which its spatializing tendency inserts), or whether this atomicity is already there in nature and is merely passively apprehended in the percipient event, we are not told. Sometimes Whitehead seems to hold that objects are such because we "factorize" or "canalize" certain of the characters of the passage, and that therefore matter is a product of an act of intellectual abstraction.[17]

Some philosophers take this difficulty as proof of their contention that we must assume that in the midst of flux, change and relativity there are implied certain ultimate, absolute and invariant principles. The absolutist will argue that without assuming that the individual's system of ideas

[16] "Physics and Perception," by Bertrand Russell; *Mind*, vol. xxxi (1922), No. 124, p. 479.

[17] *The Concept of Nature*, p. 125; *The Principle of Relativity*, p. 73.

of, and responses to, the external world represent the historical unfolding of an absolute order, which is progressively articulating itself through individuals, who give the best clue to the interpretation of the universe as a whole, the philosopher must succumb to a relativity of truth which can only end in scepticism. This position is argued in detail by Doctor Leighton in his *Man and the Cosmos,* where, especially, he asserts (page 472) that *"unless nature, in the sense of the space-time world, be a mere collective name for an absolutely discrete and chaotic succession of finite events or durations, it is the manifestation of a permanent or supertemporal order, an invariant principle."* There may be an ultimate, invariant principle implied in the diverse groups of transformations within the universe, but in a creative system still in evolution it would appear to be difficult to define what this absolute principle must be. The main respect in which the present view differs from objective idealism lies not in the denial of an intrinsic structure and order in the universe, but in the rejection of *de novo* monads, as *life* and *mind,* which are miraculously catapulted into evolution *ab extra,* and from this virgin birth forward are regarded as legitimate explanatory concepts. Emergent qualities are perhaps miraculous, but they are continuous miracles. Since creative monism is a monism of action we are bound to try to fit "life" and "mind" into the system of durations as, perhaps, macroscopic rhythms, which demonstrate their qualities or behavior in the durations in which they live and move.

V. A Monism of Action

The problem we now face is that of explaining how, why, and in what sense "emergents" emerge. By way of approach, we will retrace the conceptual-empirical founda-

tions of nature which we are taking as the source and origin of that which appears as the resultant of the world's evolutionary travail.

The stuff of the universe of creative monism may be called *behavior-stuff*. Reality is a system of events, which is pregnant with higher complexes because of its intercourse with time, the dynamic and epigenetic aspect of nature. This means that we must reject the idea of naive materialism that space, time and matter are objectively independent entities. Space involves both time and "matter." Time involves both space and matter. Matter is implicated in both space and time. Since we are rejecting the notion of substance as a self-subsistent underpinning of the phenomenal universe it is well to pause a moment to justify this view.

Matter is conceived to be composed of nodes of permanence amidst contemporaneous change. By insisting that nature at an instant is an abstraction, and that any thing, such as an atom of hydrogen, requires a minimal duration to *be* hydrogen or to display its properties, Whitehead has shown how the wall between substance and change is to be broken down. "It is the healthy thing," as James says, "to leave off grubbing underground (in the realm of the transempirical) for what effects effectuation or what makes action act." *Action or behavior-stuff is its own excuse for being*. This view also tightens the nexus between quantity and quality. Behavior-stuff is a quantity of quality. Thus we are led to cast out the category of the thing-in-itself, the hard, round atom of naive materialism, and the distinction between primary and secondary qualities as having, respectively, an objective and a subjective status. Neither is the electron a self-sufficient entity. It is the dynamic center of relations which pervades the spatio-temporal field in which it acts. Like a mathematical point, it is the limit of a converging series (or a "boundary singularity of the

field," to use an expression from Weyl's *Space-Time-Matter*), which, at the other end, extends out into externality.

A thing is not the static sum of the abstracted elements into which it is analyzed, but it is the dynamic synthesis of the inter-related elements of a behavior-complex. As a dynamic network of relations it includes all the laws of its potential and actual behavior. A thing is not only what it does, but what it is capable of doing; it is the systematic (logico-dynamic) totality of its ordered qualities or forms of behavior. The synthetic products of nature are not static entities, as they would be if their unity were hypostatized. Accordingly, life and mind are not trans-empirical entities, but they are their modes of behavior. All synthesis constitutes a dynamic equilibrium whereby differences of potential are created. In Hegelian terminology, the synthesis is a union of interacting opposites. As H. Wildon Carr[18] puts it, "A field of force is essentially the concept of opposites kept apart and held together in a state of equilibrium." As he further states (p. 284), "it would be difficult to name a more perfect illustration of the concrete universal of Hegel than that offered to us in the modern electrical theory of matter."

It is because we find relatively permanent syntheses or equilibria that the vitalist and animist asserts the presence of new kinds of entities, but they are here discounting the fact that even the most unified and integrated living or thinking individual is an unstable synthesis. Vitalism and animism are really survivals of the substantialistic view of nature—the tendency to make *things* out of *functions* or *modes of behavior*. "Substance" is only a resting-place in thought. What is a substantial level referred to one kind of behavior may be an energy level referred to another stratum of action. This is confirmed by modern tendencies in physics which construe "matter" as "energy-knots." The

[18] *A Theory of Monads*, by H. Wildon Carr, p. 290.

dualism between matter and energy appears as a special case of the more general psychological activity whereby we pragmatically adjust ourselves to external nature.

In consonance with this monism of action or *universal behaviorism* (as we might call it, if behaviorism were not so intimately associated with materialism) we may view both the organism and a material particle as stress-centers of relations which radiate out into the environment or the field which is responded to.[19] The total field of response, with its varying densities, suffers continual readjustments because the center of action and the field selected out are not in equilibrium. Some would here object that it is not correct to say of the electron, as we say of the organism, that it selects out the objects to which it responds, because we do not attribute self-activity to an electron. The reply is that this is true only in the view of matter as held by naive materialism. In our view, matter and the organism modify their environment as much as the environment modifies the storm-center upon which it acts. The difference between an electron and an organism is that as the center of interaction increases in complexity it establishes more numerous and more complicated interconnections with its environment or externality. The differences of potential which characterize the phase systems of higher equilibria render it more difficult to preserve the integrity of the dynamic unity. The reason the organism and the environment are not in equilibrium lies in the fact that the organism is a locus of energy changes which have a history, i. e., it is characterized by what Russell,[20] following Semon, calls *mnemic causation*. Russell gives no explanation of the origin of mnemic causation. He is unable to account for it because he starts with a universe which has for its stuff

[19] I have taken over some of the terminology from C. Lloyd Morgan's article, "Behaviorism," in the supplementary volumes to the *Encyc. Brit.*, last ed.

[20] *The Analysis of Mind*, Lecture IV.

logical entities devoid of the characteristics, discovered in the empirical world. The present view does not face this difficulty because we start with a behavior-stuff which accumulates mnemic effects as it goes along.

All historical entities have an accumulated reservoir of action, which varies with the history (mnemic accumulations) and complexity of the individual. An organism is the most complex system of accumulated energies, and its responses are therefore more marked by mnemic effects. An organism is an equilibrium under stress. The stimulus acts as a trigger-effect in releasing the tensions, thus giving expression to the stored-up energies. The simpler the unit the less is its responses marked by the effects of past history or experience, and the shorter is the minimal duration within which it behaves and has its being. The limits of response are circumscribed by the radius of relations obtaining between the individual and its field of externality. This is largely a matter of the inherited complexity of internality, though, of course, the higher the storm center of behavior the less is it dominated by the immediate environment. "Intelligence" is this ability to act within a wide range of spatially and temporally remote considerations. In this manner the central segments of the highest reaction level of neural arcs function relatively independently of the sensory and motor segments of a thinking organism.

The above view throws some light upon the perplexity why activity should apparently rise higher than its source. The inherited and acquired mnemic accumulations or engrams of an organism frequently put it at odds with the immediate environment in which it may find itself. Activity is the result of the tendency towards an equalization of potential, and the only reason activity, like water, does not find a static equilibrium is because there are factors which oppose it. Internal conditions tend to equilibrate themselves with external conditions, but the external conditions

of one thing are their own internal conditions. This is what makes harmony in a complex society so hard to secure. It is also probable that the individual may make differences of potential simply to give an outlet to the damned up energies which seek expression. Where the environment calls for a greater reserve of energy than is actually available the dynamic synthesis may break down into clusters of impulses or neural sets functioning relatively independently of each other. In the cases of neural fatigue responsible for multiple personality the individual may function on phylogenetically and ontogenetically lower levels of response requiring less energy.

The mystery the attempted explanation of which is responsible for the introduction of life and mind as non-biological entities is the unity of the organism and the unity of mind. These unities seem irreducible and therefore independent of their material embodiments. Thus mind is unique because its purposive and anticipatory activities seem to transcend physical space and time. Now if we can get organic complexes unified naturally, so that the whole acts in the parts, we have "explained" the mystery which is claimed to be solved by the introduction of non-natural entities. This is the problem of integration.

We may begin by postulating that the world's ground contains a vector quantity. In other words, even for the physicist the movement of behavior-stuff must have direction, for time itself, through which energy is integrated, is irreversible. To put it logically, temporal and causal relations are asymmetrical. We may here digress a moment to examine the notion of order.

The common denominator of the activity of the cosmos is order. But there is no such thing as order-in-itself. Order manifests itself only in ordered existents. Order is dynamic. It is in the temporal processes that nature rises above mechanism. Progress is impossible in

a mechanistic world because time does not "bite" into it. For the advocate of the principle of relativity, the geometric properties of the four-dimensional continuum express the intrinsic relational order of the physical universe. Doctor Alexander's[21] theory of order as arising from the relation of "betweenness" attaching to the spatial and temporal units is consistent with the relativity theory, only that time appears to occupy a more important function than that of being but a fourth coördinate of space. Doctor Alexander's theory is also significant in suggesting the origin of order as the relation between universals and particulars. Universals are spatio-temporal patterns or habits of motion which are actualized in sensible particulars, which, in turn, are spatio-temporal complexes. Universals are the living formulae of generation or the constitutive plans of things whereby Time brings forth particulars in a Space which can receive them. We may note in passing that this view implies that the laws of epistemology are laws of the universe as a whole, and is of assistance in establishing a monistic theory of knowledge. What we are more directly concerned with in stating Alexander's theory, which grounds order in Space-Time, is that it illustrates the point we are here insisting upon, namely, that ultimate reality is coherent and not chaotic. However, as we have previously affirmed, Space-Time alone is too tenuous to constitute a behavior-stuff, even though it be labelled "motion-stuff." Space-Time without matter does not exist. In line with this view we might more appropriately say that universals or forms are to be conceived as Platonic ideas functioning dynamically as energy patterns.

It is the integrative action, or tendency towards "mutual aggregation," as Royce calls it, which is responsible for the creation of more complex units. In answer to the question as to how this aggregation into more complex contours is

[21] *Space, Time, and Deity*, by S. Alexander, vol. I, p. 263, and vol. I, Bk. II, ch. III.

accomplished, several replies are possible. It may be that the complete understanding of the relations obtaining between electrons and protons will supply the prototype for all subsequent integrations. It may be, as Alexander[22] surmises, that even at the lowest level nature has adopted a trial and error method, and that the existing material elements represent a survival of the fittest. Again, we may observe that the synthetic processes seem to have predominated because it is matter which is integrating into more complex forms which thereby constitute the larger and hence more obvious forms, whereas matter which is dissimilating tends to become invisible. But none of these answers seem entirely adequate. The suggestion presented in the following paragraphs is more consistent with the general view herein set forth, though it is probably less plausible than the above possible explanations.

The universe is a self-differentiating totality or integrating multiplicity. Within this system of systems finite qualitative energy complexes or behavior-stuffs can be isolated as abstractions from a universe which includes all special systems. The synthetic aspect of nature is one side of a process which includes integration or assimilation, on the one hand, and dissimilation or disintegration on the other. A physical thing is thus a moving compromise between opposing activities. It is an invariant which retains an integral value (remains relatively constant) through a given group of transformations. Referred to another frame of reference or group of transformations it ceases to be a thing. The absolute frame of reference is approached as the center of relations encloses more and more universals. This means that the locus of the absolute is not somewhere out in the interstellar spaces. The opposition between the absolute and relative, finite and infinite, the necessary and contingent, is a false antithesis, or, rather,

[22] *Op. cit.*, vol. II, p. 55.

they are complemetary aspects of a concrete universe which includes both terms. In some sense we are living in a spherical universe in which one movement represents the return of what appears to be a counter-movement. One might claim physical justification for this view on the ground that the two great generalizations of science, the principle of least action and the second law of thermodynamics, are amalgamated in the theory of relativity.[23] It would carry us too far astray to try to determine in what sense there can be progress or emergent evolution in such a universe, but Bradley's well-known argument that the universe contains change and histories but itself has no history is no more incredible than many other true paradoxes. But it is time that we return from this excursion into the fifth dimension and apply ourselves to more empirical problems.

If we conceive electrons to be nodes of permanence at which cross and irradiate again into space (or the field of energy) lines of force, then, perhaps, increase of complexity results as a property of the pressure from the circumambient spatio-temporal field which converges towards centers of activity. This pressure may, if identified with the temporal aspect of nature, be conceived as welling up from a direction at right angles or perpendicular to the perceived physical universe. Time is then the tempo of emergent evolution. But whatever the final explanation may be, we may set it down that nature somehow arrives at more complex configurations, stable enough to generate new configurations of greater complexity. Physical integration and disintegration are paralleled at the level of thought by analysis and synthesis, which are aspects of one activity, the movement of thought along the time axis. Definite bounds are set to correlative activities of things and thoughts at the points where opposing curves of action

[23] *Space, Time, and Gravitation*, by A. S. Eddington, p. 149.

intersect and neutralize each other. On the physical level this is illustrated by the interference of light waves.

Just as creative monism regards living organisms, not as complexes played upon by *entelechy* or a vital force, but as a dynamic equilibrium having the emergent quality of livingness, so mind must be regarded as a form of behavior characteristic of organisms of still greater complexity. Many people object to the statement that mind is a form of behavior because it suggests mechanistic behaviorism. This objection can be partially overcome by viewing mind as a form of neural behavior with its own qualitative differentia, the laws of which are not the same as the laws holding for other organic tissue, such as muscle or connective tissue. Originally, of course, all organic tissue has emerged from protoplasm. The point is that the appearance of neural tissue out of a less differentiated protoplasm is no more and no less mysterious than the appearance of "life" or "mind." At a lower level, the photo-synthesis of chlorophyl is just as much a creative novelty as is the synthesis of protoplasmic systems. There are not only two big discontinuities in the evolution of nature, but there are as many kinds of emergents as there are discoverably diverse modes of activity. The retrospective analysis of science does not "explain" anything, for a thing is its behavior, and in a historical universe the cumulative impetus of emergents goes beyond the descriptive equations of physics. It may be that there are critical or saturation points in evolution at which the accumulated conditions crystallize into products which, as effects, seem out of proportion to the antecedent and attendant circumstances. This gives rise to a further problem to be solved.

The puzzling thing is that in some aggregates we seem to have not only an increase of complexity, but the emergence of a new simplicity. The most obvious cases which may be cited are the crystallization of salt, which can not

be described in terms of the properties of either sodium or chlorine, and the formation of water, which seems to possess something more than the sum of the properties of oxygen and hydrogen. But the same thing happens in other fields. If it were nothing but a matter of adding the properties of the individual components, we should be able to predict by some sort of biological integral calculus the properties of living matter from a knowledge of the properties of non-living matter. Against this contention for "uniqueness" stands the fact that Mendeléef predicted the properties of the then undiscovered elements on the basis of the position of the blank spaces in the periodic table. But even here it must be added that this predictability rested upon a previously empirically observed correlation between chemical properties and position in the table. It seems that we must simply accept the fact that the resultants of natural syntheses frequently overflow mathematical formulae. Nor can this seeming production of novelty be ascribed to the limitation of human intelligence, for we must accept normal experience at its face value.

It is probable that the presence of an organism sometimes increases the apparent synthetic novelty of a perceived feature of nature. Thus Kant held that the source of all synthesis was in the experiencing subject. As an illustration of a possible projection of a novelty into nature which may have its origin in the perceiving organism we may cite the following: We differentiate between hot and cold bodies as though these attributes referred to qualitatively different external realities. But for physics hot and cold are not two separate things; they are but two points in the temperature gradient which ranges from absolute zero up to the highest temperature obtainable. While the physiology of sense perception is not fully understood, we know from objective study that we have two receptors for temperature, one for warmth and one for coolness. Per-

haps, then, it is because the two different sense organs give us two different sets of sensation, that we have come to project this difference into the external world. Considerations of this sort, as we have seen, have led some who have reflected on the matter to suspect that qualities are not really in the external world, but originate in the organism. In keeping with this conviction those who affirm the subjectivity of secondary qualities might argue that the quality of "wetness" which emerges when oxygen and hydrogen combine to form water is not a "unique" quality, but may be analyzed into the various psychological factors involved in the tactual experience of wetness—such as temperature, decrease of friction in moving "wet" fingers across each other, etc.[24] The baffling thing is that water has not only psychological but also certain physical-chemical properties not found in its constituents. Water has surface tension, specific gravity, is subject to capillary attraction, etc., and these are not *a priori* predictable. Certainly, we are here face to face with one of the most accute problems of thought. The only thing I am certain of is that the mind contributes nothing to the external world, even though qualitative differences may be increased by the presence of a perceiving organism. If this is what Whitehead and Alexander mean by the objectivity of qualities, then the present account is in agreement with their position in this matter. Bertrand Russell's formulation of the problem, wherein he holds that the laws of the intervening media, including the nervous system, must be taken into account in describing the appearance which a piece of mat-

[24] Professor Titchener takes this position in his *A Text-Book of Psychology* (1913, page 338), when, in arguing against Wundt's theory of space-perception, he says: "To say that space results from the fusion of quality with intensity, however plausibly the statement be made, is to leave us with a mystery; nowhere else, over the whole range of psychology, does the concurrence of attributes give rise to an absolutely new form of consciousness." But this seems to contradict an earlier statement (p. 123), "In all such cases (of smell mixtures) the resultant odour is simple and unanalyzable; it resembles the component odours, but it can not be resolved into them." Probably the apparent contradictoriness of the two statements can be dispelled by the definition of the term "absolutely" in the first quotation.

ter has when viewed from a perspective, seems to be a happier way of stating the matter, although Russell does not tell us what is at the other end of the intervening medium when a nervous system perceives a piece of matter.[25]

All allowances being made for the contributions made by the experiencing organism, there still remains an irreducible residuum of qualitativeness, which we must attribute to external nature as true features of her creative advance. This compels us to reject Holt's account of form-qualities as being nothing but logically ordered neural periodicities. Physiological densities are not densities of abstract number series, but of something, and a thing is qualities functionally related. Qualities are behavior-stuff. Logical order is an abstraction from the temporal order which unifies the simultaneous qualities and successive states of a thing and its permutations. It is through the emergence of the unpredictable serial order of creative novelty that nature transcends the scheme of mechanistic materialism. Once the view of creative monism is established in the place of mechanistic atomism, against which vitalism and animism stand as a protest, the dismissal of meta-empirical entities as explanatory concepts will be in order.

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[25] *The Analysis of Mind*, p. 116.

THOMAS THORILD: A CHARACTERIZATION[1]

(Translated by Arthur Julius Nelson, M.A.)

STRICTLY speaking, the numerous philosophical works which Thorild wrote in foreign tongues, especially those which he wrote in German, can never be purged of many serious defects, but nevertheless it is surprising that they should have been so completely forgotten. This, in some measure, may be due to the errors in language which occur throughout his works. In the nature of things, Thorild's gifts as a stylist and author could not be utilized to their best advantage in his German writings. In them his struggle with extensive and stubborn material is revealed by the very incomplete manner in which his powers as a writer have been realized. Up to this time he had never been called upon to make use of this rich and modulated language, and therefore, it is only natural that he did not

[1] Thomas Thorild, Swedish poet and philosopher, was born in Suarteborg, April 18, 1759, and died in Greifswald, October 1, 1808. An orphan, he studied at Göteborg, then at Lund where he met Lidner. In 1781 he went to Stockholm and founded a review, *Le Nouveau Censeur*, in which, as a result of the insufficient academic recognition given to his poem, *Les Passions*, he attacked vigorously the literary and political prejudices of respectable society and of the Academy. At Uppsala he presented a brilliant thesis, *Le Critique de Montesquieu*, and went to England (1788) where he published in English The Sermon of Sermons (1789). In 1790, having returned to Stockholm, he resumed his polemics against Kellgren in his *Critique des Critiques et Projet d'une Législation du Monde de Génie*, the best of his works, wherein his broad views and enthusiasm are mixed his naive pride and matchless eloquence. His work, *Sur la Liberté du bon Sens*, gave his enemies occasion to condemn him to four years exile. Thorild then went to Copenhagen and Lübeck, where he published his work on *Le Droit* (1795). Appointed professor and librarian at the Swedish university of Greifswald, he there composed *Maximum seu Archimetria* (1798) then *Die gelehrten Welt*, letters to the French Academy, to the Pope, and to the Czar Alexander, collected under the title: *Orpheus sive Panharmonion*, all of them remarkable writings in which one can measure his high worth as a philosopher, although they are not widely known. (*La Grande Encyclopédie.*)

succeed in handling it with the best of skill. To be sure, there is evident in his German writings a strength and a certain aptness for hitting upon the unique and suggestive word, but the finer shades of the language's rhythm he does not seem to have mastered. Monotony, which he affirmed so often was the most detestable weakness in a writer, just as liveliness of style and melody was a writer's greatest virtue, creeps into his own short, robust sentences with a repressive and deadening effect. To be sure, we do meet with some clever and captivating pictures; but the composition is weak, the form itself is not as faultless as it used to be, while the finer subtleties of artistic expression are wholly lacking. These defects of beauty and this barrenness of style are all the more striking if we read his earlier Swedish works; the clear, swift diction and brilliant rhetorical effects of which give them a little of that dash, the *sterneana protervitas,* which, somewhere, in his letters to Cramer, he professes to scorn. What particularly spoils his German writings is a tedious toying with euphony, a weakness for puns and paradoxes and a labored striving for far-fetched comparisons. Indeed, it seems at times as if he intended all of these verbal extravagances and sins to serve as effective means of directing public attention upon himself and his works.

We get an entirely different impression of Thorild's chief works which were written in Swedish between 1780 and 1790. In them his mastery of style and form is revealed in all its inimitable charm and audacity. This portion of Thorilds' writings possesses grace and gaiety, and a lively rhythm which catches us involuntarily, and makes his prose some of the most subtle and musical that we have in our whole literature. Or, indeed, we may be captivated more by his frankness and spontaneity, his fiery persuasiveness, his zeal and integrity which give to his work the character of a confession; a complete, unified life picture of the

man. Nowhere in the Swedish writings can we find a work of art standing detached from the author; each has an obvious and direct connection with its creator. Indeed the more deeply we delve into his Swedish works, the more insistent becomes the urge to go farther and farther; and yet, in the end, we find it impossible to absorb all of one who lived by himself in a complete and detached world. Thorild does not create independent living characters; and, on the other hand, what he has experienced and absorbed of life he does not mould into fantastic beings. We can scarcely detect any trace of character sketching, while in those instances where we might term his characters fictitious, these are either clothed in complete idyllic unreality or are lost in an intellectual chiaroscuro. On the other hand, he frequently touches off a scene with few strokes, and occasionally, as in his letters, he sketches clearly, sharply and realistically for us an interior, a landscape or a detail of his daily journeyings and travels. But in a strict sense, Thorild does not describe. He seldom notes the outward or bodily aspect of things, and details interest him less than do theoretical meanings and outlines. His mind is set rather on the larger, rhythmical contours of life and nature, or as he himself says, "on the pathos existing in nature, especially that part of it which is born and dies; the pathos of dawn and twilight; of light and darkness; of youth and age; of spring's mild sun and autumn's frost; of faint autumn rays and falling leaves and withering flowers." [2]

Thorild's nature poetry does not lack occasional heavy shadows and strong lights, but harsh accents are on the whole rare. His lines of this type are often struck off in the traditional light tones of silver and rose, and their airy, pastoral beauty gives them a peculiar, uncommonly delightful effect. Something esoteric, something elusive and spiritual hovers over his words. A veil of misty dawn rests on

[2] *Thorild's Collected Works*, published by Hanselli, I, p. 280.

his verses, hiding myriads of white and golden hues, and faintly suggesting the cool, pure fragrance of spring. When Thorild studies nature, he does not, strictly speaking, concern himself with her outward forms. He does not seek to reveal and express any hidden, ingenious connections between the multitudinous outward forms of existence or their hitherto unnoticed inner relationship. He gives no vivid, minutely exact description of a section of landscape which he has carefully observed, nor of a plant or animal which he has patiently studied. Instead he forces us to feel rather than to see the object which impresses him, so that what remains after everything has passed, is the memory of perceptions and feelings rather than pictures. Nature to him is a living being which gives strength to his own light, energetic optimism. He sees it steadily through a metaphysical sheen, animates and spiritualizes it, endows it with the attributes of human life, gives it endless perspective and a rich symbolical meaning. Nature in his eyes is of the same stuff as himself; it mirrors from beginning to end his own expansive being and overflowing soul-life.

There is a modern theory of aesthetic criticism which holds that a person should be quiet and expectant before a work of art, should confine himself to an understanding and an analysis of the impression and inspiration he derives from it, and then should interpret the modification which his nature undergoes by virtue of it. When we harmonize this theory with the elementary and slightly detailed criticism which appears here and there in Thorild's work, there stands forth clearly the great difference between our contemporary flexible and artistic methods of criticism and the far less proficient and subtle type of criticism which Thorild represents. But notwithstanding that, we may have to admit that the sound and comprehensive method of criticism set forth in the above words is, in one respect,

more minute than Thorild's. Such a theory as the above one would undoubtedly strike him as altogether too refined and artificial, and too passive also, perhaps. To him it would appear as an unnecessary round-about way of dealing with creations of art, going foul of what he considers to be the deepest and most vital part of them. The theory which he himself brings forward in this connection certainly tends toward that same concern for literary and aesthetic analysis which we have just mentioned, but at bottom it has meaning very much different from the former and is far more dynamic. To Thorild, poetry is essentially action, and falls almost entirely in the domain of ethics. He is an implacable foe to the precept, art for art's sake. He is less the artist than the craftsman, and hence we should not deal fairly by him if we essayed to judge him entirely by aesthetic standards. Properly speaking, Thorild is a layman in the field of the arts, and does not wish to be counted as anything more. He is a consciously active and ethically guided individual as well as a poet. In response to Leopold's question: "Why do you persist in making literary enemies?" he writes in one of his letters: "Because I want non-literary friends. I seek what fascinates me: noble and pure friends, worship, praise of the soul, natures' people. Is not this the only true and eternal honor? The censure or praise of the ignorant or the superficially clever amounts to nothing. They are mere ripples in the mainstream of life." [3]

In his poetry Thorild looks upon a phase of human character as unworthy or offensive according as its emotional contents, its innate transforming power is capable of allowing for cultural growth and development of personality. Hence, there creeps into his poetry a marked tendency to didactism and moralizing, even though he is too much the artist to miss the joy of the beautiful in the creation and

[3] *Thomas Thorild's Letters*, published by L. Weibull, p. 116.

perfection of his theme. His later poetry, on the contrary, bears little trace of that poetic creativeness which springs from an urgent and inward impulse, and which takes form because of this organic urge and impulse. In it we do not always catch the inmost and deepest secret of man's soul, nor does his poetry always convey the impression that he has sought out a solution to the problem of love. But in spite of this lack of intimate insight which is noticeable here and there in the creative process, we cannot put him in the same class with the artist who has but a superficial understanding of human nature. On the contrary, in his mind the symbol and the thing symbolized stand in intimate, mystic relation to each other, and it was foreign to his nature to analyze away this relation. From a philosophical standpoint he sees no difference between the word and the object it denotes. "The soul of a word lies in the feeling it evokes, and this feeling is indistinguishable from the feeling that is aroused by the thing which the word denotes."[4] And foremost among poets he places those who "do not play with but live in" the word-pictures which give language its immortality. In the field of aesthetics he does not essay to keep apart artistic expression and the complex of feelings which lies back of it. "So far I do not understand," he writes in a letter to Leopold, "and to the extent I do understand, I do not like, these differences that are made between symbols and the things symbolized; between form and feeling. I find unity in all nature. What is sublime in a picture is sublime by virtue of the emotion resident in it; an emotion is sublime only by virtue of the truth it possesses; as for example,

So Atlas' lordly summits
Pierce the vault of Heaven
As the traveler with uplifted eyes
Amazed, trembles at its base.

[4] E. Reinhold, *K. L. Reinhold's Life and Literary Works*, p. 284.

Is it the object itself which excites here? Or is it rather the endlessly beautiful and pathetic suggestion of the stupendous and insignificant which moves one to awe?

> On the brink of death
> Or danger's roaring chasm

is not a mere schematism—it is a truth taught not in school, but in living nature itself:

"Let us separate things according as they are true or false, large or small, strong or weak, etc., so that we may not preclude anything which nature does not preclude."

In actual practice Thorild, in certain portions of his earlier works, appears to regard poetry in its highest form as the most complete and everlasting expression of truth and reality we have, and in his mind it cannot be judged by standards less universal than those we apply to expressions of human desires and motives. The three basic laws which he sets down in his "Criticism of Critics" as governing literary criticism may be said to have this characteristic of universality. They stand as independent points of view and the connection between them is by no means purely theoretical, logical and deductive. What they have most intimately in common as their goal, and what they fundamentally enjoin, is a certain code of justice, a humaneness and a feeling of moral responsibility which Thorild considers should be the characteristics of all types of criticism and evaluation. Even though we will admit that a moral idea, sufficiently developed, coincides in the end with an aesthetic one, I believe, nevertheless, that we must own that Thorild occasionally measures literary productions with a moral rule that is altogether too severe and inflexible and thereby does not allow the purely artistic virtues of a given work their full due. But by virtue of that very feeling for right and that broad sympathy to which his judgments and evaluations bear witness, his laws retain

to this day all their vital power and direct applicability. Without a doubt these laws deal with fundamentals and possess undeniably a high degree of universality, which leads to another of their characteristics: they are with one exception almost purely formal rules similar, in some respects, to what Kant's moral maxims and categorical imperative are in their field.

When Nietzsche somewhere in "Jenseits von gut und Böse" is explaining the formulation of metaphysical propositions and the erection of philosophical systems, he says that the moral or unmoral ideas in every philosophy constitute the real seeds from which the system sprouts forth, and that it is therefore always safe and wise to ask first on what moral idea the whole system is based. This method of criticism, which does not lack possibilities of application far outside the limits of philosophy, is aimed, like Thorild's principles of literary criticism, exclusively and directly at the inner meaning. The basic idea in the methods of both men seems to be the same, but their resemblances do not extend beyond that point. The former furnishes primarily a method of exposition, the latter a method of evaluation, and these two methods are allied only to a certain extent, never wholly. Nietzsche's principles of criticism especially have a psychological tinge and show evidence of a subjective keenness in which we see no resemblance to the objective standards of Thorild. Although Nietzsche saw something pernicious in the erection of such a starting point of criticism, it was this very perniciousness which gave it value to him. Unsparing analysis enticed him, for through the use of it he saw the possibility of attaining new and unexpected results. Thorild's aesthetic sense is never troubled with such theoretical impropriety. This sort of intellectual malice and this insidious unreasonableness in searching out places where a writer exposes himself to accusation, is not characteristic of him. Thorild's own

criticism does not lack its objective severity and polemical keenness, but nowhere in his criticism is there evident any of that heartless and inquisitive psychologizing which Nietzsche makes so much use of; nor is there to be detected in it any trace of that harsh and unscrupulous *médisance* which has often played such a significant rôle in the best works of our criticism.

On various occasions Thorild pleads for the establishment of an associate chair or a special faculty which shall take precedence over all others and which shall see to it that neither the other faculties nor the life of the community at large sinks to a level of mediocrity.[5] It should serve as a kind of examining office for theory and practice. In the same way exactly does he look seriously upon his criticism, which extends beyond the limits of the purely literary and aesthetic. The whole of cultural life appears to him as a department of legal jurisdiction, and the office of critic to him is like the office of a judge. As he himself says, criticism implies nothing more than "the administering of justice in the world; being, in a high and noble sense, just." Therefore, he desires to woo and win human society's highest, rarest, most wonderful law, justice. Inwardly he strives to attain the highest measure of objectivity; certainly he never overlooks the outward form: the method of exposition, the vigor and sharpness of style, the whole artistic arrangement; but his judgment is never swayed thereby. To Thorild, criticism is an evaluation based upon definite moral-aesthetic principles rather than what it has come to mean in modern speech: a commentary or mere reviewing. For that very reason our interest in his critical works is kept alive. Here as elsewhere his writing acquires force through the mysterious crystalization of a poetic temperament and a unique personality.

[5] Cf. *In Memory of Thorild*, Appendix I, Sec. 150.

Even as a philosopher and as a student of social-political questions, Thorild is less an inquirer, less a thorough and patient investigator than he is an appraiser, although at times he does startle us with a capacity for detecting subtle psychological and cultural traits. His special works, therefore, can be regarded in their entirety as so many different chapters in his life's confessions, and to that end his letters also form an illuminating commentary. In no other place do we find him more directly personal, and nowhere can we get closer to him. Nearly every word gives the impression of having an intimate, clearly constituted connection with the man's speaking self, and possesses a personal touch which inspires confidence and conviction.

It happens occasionally that when a man seeks soberly to account for his judgments and conclusions, we find out something about his organic life: its hurried pulse-beat, and its complex processes. Then it is that we seem to get, more often than at any other time, a clear idea of the conditions which govern the weal or woe, the strength or weakness, of his life. We try, above all, to search for those manifestations of his organic life which allow us to trace the origins of the conditions mentioned, or which enable us to discover the inward and outward experiences which have little by little deepened and reformed the river-bed of his consciousness. Poetry is a shimmering and many-colored dress which both hides and expresses without enabling us to decide positively which of these functions it is performing at a given time. What we are interested in is to get behind the life lived and suffered by the man we are studying. Particularly are we anxious to look for those instances when he has been face to face with crises so that his message may be somewhat clearly defined and so that we may get some inkling as to the driving forces and leading motives which have most profoundly and effectively influenced him. For this purpose, Thorild's letters are the

most valuable documents we have. Nearly every page deals with realities of the sort we have mentioned. The earliest ones of which we have any knowledge possess a decisive and fundamental importance for a thorough understanding of his character. They tell of his conduct as a student at Lund, of his intellectual equilibrium and of his passionate struggle for spiritual independence. One can see how the university's pedantic atmosphere and its unfree social life with cliques and little ceremonious distinctions makes the youthful follower of Rousseau so discontented that he feels completely homeless and is unable to hold out any longer. The people with whom he comes in contact are small and inferior. "They do not understand how to share each other's joys and favors." He becomes absorbed in conversations with his trusted friends, and more than one of his letters to them contain confessions of frankness and sincerity beautifully expressed. Interspersed here and there are brief notes and descriptions which from a historical as well as biographical standpoint, are of no slight interest. In his correspondence with Anders Hylander, there is one striking letter wherein he gives highly passionate expression to the lonesomeness which seized him while temporarily separated from the friend of his youth. "I watched your carriage disappear. At first prevented by my companions from feeling your loss, I arrived home at last. I wanted to read, I wanted to think, but I could not concentrate. Those tender and longing glances which you cast as you beckoned me farewell from the carriage and which you yourself saw made me embarrassed, arose unceasingly in my mind's eye. I felt my heart overcome by tenderness and grief. Then it was that I sought freedom and solitude. Arist! Shall I tell you all that happened? How I wandered about, sad and solitary, calling out your name and shouting how much I loved you? In vain, I

cried, do I flee. Dear Arist, my thoughts follow you, while my heart grieves that you are not here."

What he missed through the absence of his friends is disclosed in the following words: "Alas, Arist, none of my friends are beings after my own heart, as you are, for none of them have joys or sorrows that are so like mine as yours are. I have not had one taste of the sweetness of a free and tender trust, since you went away. Arist, with whom shall I now converse about philosophy and love? With the simple-minded, who have no just and honest adoration for truth, who have never grieved for her, never given her an offering, never invoked her divine grace? Or shall it be with the proud and hardened minds who set themselves up as tyrants and despots in truth's kingdom, who threaten with chains and everlasting torture and see nothing delightful and good outside themselves? No, Arist," he exclaims, "few people love truth as you do; they worship only their own thoughts." [6]

In this little document of warm unfeigned friendship, there is a great deal that is conventional in both form and feeling; but there is also in it an undertone of unmistakable genuine sincerity which engages and convinces and distinguishes it from the usually ornate letters that were written in that age of gallant expression. There is no mistaking it—in the midst of an age of elegance and coquetry with its overflowing spirituality and its languishing emotionalism, we perceive a new force, a kind of emotion hitherto not to be found in our literature, a profound personality with real affections, undisciplined in places, but powerful and deep and incorruptible. The impressiveness of this new and profound nature is heightened by reading his farewell letters to Heurlin, one of Thorild's most diligent correspondents and faithful disciples at this time. In the course of their relations, Heurlin received impressions

[6] *Thomas Thorild's Letters*, pp. 7-9.

which stamped themselves on his character for the rest of his life, and experienced a broadening out of his powers which amounted to a complete conversion before he finally attained his master's own buoyant nature optimism. Some of these letters are the most soulful and beautiful that he ever wrote. They show evidence of an almost incomprehensibly acute perception of happiness, of a complete and deep organic harmony in his nature. A stimulating and liberating force pervades them all. Nothing in his eyes seemed inchoate or difficult to understand; every problem could be penetrated, everything had meaning and importance. Life appeared to him as an eternal feast dominated by never-ending cheer; its dark sides and disharmonies, sorrows and evils did not subtract from the positive nature of happiness. Throughout the greater number of his letters, which afford excellent proof of Thorild's metaphysical prose-lyricism, gleams a steady light and over all of them rests the transparent atmosphere so peculiar to his passionate spirit.

"How I wonder if you have tasted of freedom; not of the futile fanaticism of the fanatic, but of true freedom; the unhindered and harmless play of our natural faculties, understanding of the truth, love for what is good, and appreciation of what is sweet. How noble and edifying is its feeling. It alone makes nature worship the sublimest of religions. Through freedom the soul soars aloft to the heavens as though on the wings of a guardian god. It looks down upon humanity swarming below. It gazes upon it with both scorn and sympathy. It sees fools, heroes, kings, children. It beholds the wise man in his mountain cleft, lonely and forgotten, but noble and full of the spirit of God. It knows that everything is perfect and blissful, that everything is as it should be—the diversity of life unfathomable. It is filled with high seraphic wonderment; it knows God, and expires in the immensity of His power,

and goodness and beauty!—oh holy rapture! the arcanum of my religion—intelligent, enlightening, free!"

"Wise, noble friend! let your interest be not in humanity only, but in all nature; not in dissent, but in agreement; not in narrow nationalism, but in a broad internationalism; not in the concerns of individuals, relatives, friends, but in the concerns of the community at large. Do not stray from that which is great and whole; the noble, universal blessedness of companionship with God! Dear friend, if I could only reveal to you the eternal kingdom of nature—what light you would see, what indescribable feelings you would enjoy! No, none is happier than the wise man. Alexander, Carl, was the noblest animal that ever lived. But the wise man breathes in the life of God.

"One of nature's faults is that she did not endow the average man with the capacity to understand all things. The wise man does. Mark, my friend, this noble truth! the wise man sees with the eyes of God.

"The philosopher, the world's judge, can not be partisan. He must stifle the stupid, the unlearned, the rebellious. He must teach man the noble, comforting humility of reason. Behold the wise man sitting calmly and blissfully under his tree. Such godlike greatness! Renunciation! Through it man enters the paradise of eternal freedom, finds in suffering itself happiness and honor!

"To be oneself, to enjoy what is within one's power to enjoy, to worship nature—that is the highest wisdom. But neither Heaven nor Hell can do anything for you if you once lose your soul. O my friend, understanding, purity of heart, freedom: that is our lot, yours and mine. Divine understanding, sublime purity of heart, blessed freedom—each in its proper measure is within our reach. Let us depart with the poet Plato. See, the path lies before us.

Come, be received. We are merely two mortals in search of wisdom." [7]

Thorild endeavors, for the sake of his friend, to throw light on the contradictions in the life of nature and man. He allures and consoles, warns and encourages, gives freely of his metaphysical wisdom and pantheistic religion uttered with a conviction and fire which to this very day, willingly or unwillingly, shakes us from our self-complacency and liberates us from the bonds of spiritual slavery. A peculiarly chaste passion, beautiful and attractive, in spite of its impetuous outbursts, is to be found occasionally in his letters. In them, we become aware also of a warm sympathetic feeling for the wronged and neglected, and often, again, we note a youthful melancholy thirsting for affection which serves to make the figure of Thorild in the 1780's unusually captivating. As the years go on, this tone fades and partially disappears. The last letters to Heurlin are filled with marks of a friendship that is far more languishing and deliberate than it used to be. And to a still greater extent his letters of acknowledgement to Peter Tham strike me as real masterpieces in the expression of calm and sovereign gratitude. The concluding apostrophes in these letters possess all the amiability and a little of that stimulated warmth which distinguishes most of the prolific and elegant letter-writers of the eighteenth century.

Despite his enthusiastic worship of friendship, Thorild was not really fortunate in his associations. In this connection it is not exactly known what caused disagreement. It is sometimes said that it was the fault of the time that relations of this sort were too hastily formed and became too frankly intimate. Like Rousseau's associations, they were stormy at times and often broke off precipitately. They usually experienced sudden changes, and under such circumstances conflicts were admittedly never entirely

[7] *Thomas Thorild's Letters*, pp. 32, 49, 50ff., 62.

avoided. Thorild's correspondence includes many instances of such conflicts, and one of them he dwells upon at considerable length in a letter to Hylander, valuable on account of its display of youthful agitation and amusing self-observation.

Both had chanced one day to fall into a dispute concerning some ticklish problem in metaphysics, and Hylander had lost his patience. Having reached home, Thorild under pressure of the powerful feelings which overwhelmed him, scribbled down a few notes and these he communicates to us in his letters, "My God, how shall I explain this impulse to wreak excessive vengeance for an ordinary wrong?"

"It is true: my reason tells it, and my conscience sanctions it. I had a feeling of mingled indignation and contempt for the cold and torpid manner in which you treated the most vital principles of which human intelligence is cognizant. I wanted to avenge myself for the slight impression they made upon you and to mortify your egoism in order to force you to examine these principles and thereby make you respect them. Even at this very moment, however conscious I am of the indiscretion I committed, I believe that you cannot take pride in a belief the principles of which you are so uncertain about, nor can you flatter yourself on a love for truth which is so weak and inactive. Consequently I was right, perhaps, in sustaining my fundamental propositions, however keenly I saw that they disturbed you. When I consistently argued down your position should you not likewise have disposed of mine before you could assume, without appearing absurd, an attitude of self-satisfaction?

"However, let it be—I was unfair. Oh, God, this spirit of vengeance—so base and unworthy, and doubly base because it comes upon me so easily!"

"No, I shall not harm him a bit. I shall treat him with indifference.

"But I foresee how my flightiness will lead me astray. I shall have to be more careful."

"How incomprehensible! I who am otherwise so easily put out. I am not angry with him. Oh, Arist, I have already quite forgiven you. But my pride, my grievously wounded pride!"

Arist came to him in the meantime and begged forgiveness, which astonished Thorild greatly. "I cannot understand how you could do it. It is true, my heart does not experience any delicious thrill over it; it is in too violent a condition to feel anything delightful. But my reason does what my heart will not do. It finds you indescribably amiable—I respect you—with all my soul; but I need a few days' rest to recover my love. Oh! why can I not feel immediately all that I want to feel? Why do these stairs appear so detestable in my imagination and make me shudder? Why can I not fly to you as of yore, freely and with joy?"[8]

The description of an emotional state which Thorild furnishes us in this letter strikes me as significant in more than one way; above all, because it lays bare some of the unconscious, elementary instincts which formed the bedrock of his soul. The last lines especially seem to me to afford a sure basis on which to judge his personality, give us an insight into his innermost self, and serve as a medium through which we sense his secret contact with the current of life. This sense of being, as it were, physically fettered by chains, this purely bodily craving for perfectly ordered relations discloses an unusually pronounced and fundamental peculiarity in Thorild's make-up. In all his letters it is brought out with a vigor and persistence which makes his yearning appear more than a common desire for sym-

[8] *Thomas Thorild's Letters*, p. 11ff.

pathy and well-being. Shortly thereafter he writes to Florenz Stohl: "How I miss you! Every evening when I enter Scharffenberg's room and find myself among all these hard and crude characters I shudder, and long for you. Last night I happened to go down to Mr. Berg's rooms with a book which I had borrowed; I suffered real agony as I stood in the hall-way, beside the door and at the foot of the stairs—indescribable joy! gay and gladsome movements which come to me no more." [9] Time after time expressions of this kind recur: "Arist," he exclaims, "it is real happiness to have a soul free from prejudice and malice." And in another place: "Hate is my heresy, my deadliest sin." [10]

Placed against such a background many of Thorild's utterances, it seems to me, lose most of their unreasonableness and arrogance. His consistent pleading for uprightness and his passionate revulsion from the falsehoods and deceptions of society, from public and private cynicism, consequently does not descend to empty words or mere boasting; nor does it take on the character of an impersonal echoing of Rousseau's demand for uncompromising moral and intellectual rectitude. The rights that he contended for before his judges in the trial of 1792 acquire, by virtue of the background furnished us above, an entirely different and deeper meaning. After all, it was not as a cunning lawyer writing a clever and convincing plea that Thorild, on this occasion, wrote in a promemoria: "But concerning my motives which have been so plainly disclosed elsewhere in many tender and living words, let me say just one thing: when on several occasions I desired to express certain thoughts, my heart kept saying: 'that would not be honorable.' " [11]

[9] *Thomas Thorild's Letters*, p. 20.

[10] *Ibd.*, pp. 25, 19.

[11] *Thomas Thorild's Letters*, III, V. 82, Uppsala University Collection of Biblical Handbooks.

It is Thorild's acute sensitiveness which gives rise to this urgent demand for honesty: "I looked upon my faults," he confesses, "with all the disquieting and constant sensitiveness which a man experiences when gazing at the sun." And no doubt it was this sensitiveness which made him a relentless enemy of all kinds of deceit and chicanery, of stealth and secretiveness, of petty guile, of fraud on a large and on a small scale. If this hostility to such practices were not rooted in his nature, but were based instead on the ordinary everyday opinions that are entertained about them, it would not possess this subjective accent nor would it be of such vital significance to him. "The prince himself," he says, "bred with principles of highest honor, generally becomes so noble (with all his honor) that in order to really exalt him to the ideal of the divine in humanity, one need only to proclaim solemnly that absolute candor toward him is the highest of all virtues in the state, the political sublimation of character." [12] Thorild lays stress on the need of this purity of heart in every department of human activity, and emphasizes the necessity of plain demonstration and unhesitating frankness. In his political science he regards this quality as one of the leading factors in every social union and one of the fundamental requisites of all social life: "Consequently, in every society of men; that is, in the true state, where the well-being of all is seriously desired, the idea must never be lost sight of that candor is sacred."

"What the great patriot, the hero of men, holds as most important is "Offenbarheit." For secrecy breeds evil; and what the light of day is to the world, so is openness of heart and mind to humanity. The highest "Offenbarheit," the most sacred of all rights, sacred as the right to the divine light of day, proclaims therefore every attempt to sully honor and knowledge as the greatest crime in the state." [13]

[12] *In Memory of Thorild*, Appendix 1, Secs. 161, 155.
[13] *In Memory of Thorild*, Appendix 1, Secs. 152, 155.

In Thorild's view the practical goes hand in hand with the theoretical, and using this view as a starting point the question occurred to him, to what extent Rousseau's doctrines taking them together, in view of their intense emotional contents, ethically speaking, and their radical rejection of all dissimulation and artificiality, can be considered as indicating a step forward not only toward moral but intellectual perfection as well. At first view the life of the mind seems to have no direct relation to the life of the emotions, in fact, appears to be antagonistic to it; but on closer examination the results do point perhaps in such a direction. Emotionalism and impressionableness go together more or less; they are conditions which determine the richness as well as the vitality of a man's soul-life. All human beings are largely subject to the same sensations and the same mental excitation, but not all of them possess equal capacity to respond to them and to stimulate them in others. They never know what it is to really suffer or to be really happy. They are not conscious of all the forces that may grip a man's heart and compel him to outward action or to intellectual accomplishment.

To Thorild's buoyant and tender nature there was something brutal and obstructive to life in every emotional state. Somewhere in his "Ethics" Spinoza has pictured how malice makes a man unfree, and how it may blight his feelings for a long time, though he be the poorest and most insignificant individual. He looked upon malice essentially as a condition of stupidity or sickness, and it was this same negative and destructive character which Thorild attached to emotional states in general. He regarded them as rebellious and unbeautiful, imperfect and degrading. They threw him into a condition of unendurable restlessness, closing the channels of his creative imagination, disturbing his peaceful meditations, and laying waste his world of dreams. Here we sense the subjective factors which gave

use to Thorild's philosophy of conciliation; indeed, the physiological foundation, I may say, out of which it sprouted and grew. This sensitiveness determines and explains why his wisdom caused him to advance a utilitarian philosophy and doctrine of happiness. We can perceive here what it was that compelled him to accept and assimilate just those representations and ideas which extended in the direction of such a philosophy and doctrine. Out of his craving for harmony within himself, there grew and expanded the desire for universal harmony, a condition where all conflicts would cease and all differences that keep men apart would disappear forever. Thorild's philosophical works betray in every sentence an energetic striving to prove, by means of sober and calm thinking, the possibility of realizing universal harmony in word and action: "The ideal state of well-being engendered through pure love and loveliness whose action forms the dewdrops on the rose-petal and the tender light of the eye, as well as the sun of suns." [14] But even in these works the poet's views and dreams are interwoven in the solid fabric of his thought like a woof of gold and purple.

In a sketch called "Character," valuable as a psychological document, Thorild has attempted to trace the features of his own spiritual physiognomy and to account for the mainsprings of his inmost being, the impulses which were the most determinative for him. In this interesting fragment any indication of different phases of development and their relationship is lacking, but notwithstanding this, it is a little masterpiece of psychological delineation and seems to me in many ways to be thorough. For all its brevity and conciseness it serves as a very reliable study by virtue of Thorild's incomparably dispassionate and pointed way of setting forth his ideas.

[14] *In Memory of Thorild*, Appendix 2, Sec. 1, Paragraph 2. Cf. *Ibid.*, Appendix 1, Secs. 28-30 (his hymn to love).

"I had but one great, fixed thought in life," he confesses, "that was, to explain nature and to reform the world.

"Everything I thought and did was directed toward that end. I wanted to erect truth and magnanimity in men's lives. Such was my sole ambition. I felt I was either the greatest or the proudest of all men.

"So marvelous was the sensitiveness and sagacity of my soul, that a little spark gave me a fire of great magnitude, and a word shed me a light of deepest truth. The impulse to discover and to reform everything came natural to me. I was so certain of discovering everything, that I challenged my friends with some of the greatest discoveries within a given time, and did so without charlatanry. For truth, truth in nature's eternal light and truth in the meanest particle of dust was the joy and life of my soul."

"I worshipped unceasingly my own soul's unmeasured tenderness, strength, light and magnanimity. This adoration was the most sacred part of my happiness. I also believed that to be mortal was to be God in nature. I wanted to be everything in order to uplift everything."[15]

A youthful hope, a world of beautifully radiant optimism and sturdy self-reliance is revealed in these words. We sense in them an insistent urge, springing from an unquenchable love, to reform and to make happy the world. Perhaps there is evident in them a trace of immoderateness in desire and effort, and a defective appreciation of shades and proportions which compromises a little this brilliant philosopher of harmony. But be that as it may, we do have exposed in these sentences one of the main nerve-chords in Thorild's intellectual make-up, and our attention is attracted by them to those lineaments which mark the traditional Thorild figure. We catch a glimpse of the conscious metaphysician and poet; we get an idea of his highly emotional and sensitive imaginative life, his swift, abrupt,

[15] *Thorild's Collected Works*, published by Hanselli, I, p. 278ff.

romantic fancy which, according to the notions of the time, was considered to be a mark of genius. Here is also revealed the tireless re-creator and re-appraiser of values who discussed with such marvelous discrimination and devotion and with such penetrating understanding, the cultural problems of the eighteenth century; and who, even to this day, through the influence of his genius as well as his works, enjoys the same actuality and interest which he enjoyed a hundred years ago.

His original personality which is very much apparent in the above quoted words, stands out more sharply in one of his letters. Leopold had described to Thorild his dejection on reading Kellgren's criticism of his, Leopold's, "Ode on November 1, 1778." In answer to this letter Thorild writes:

"I cannot bear to see a genius oppressed or to see a noble man suffer. But I cannot love, indeed, I have deep, violent and yet tender disgust for the weakness of will which is always the cause of this suffering. If a man whose intellectual power the world is waiting to see demonstrated, by whose daring genius it is expecting to be both benefited and charmed, has not strength enough to maintain his courage—how is the man who has the task of uplifting thousands going to look upon his mission? He who has written Sweden's first ode—understands what I mean. I do not love the man who needs more than what his own soul can give him in order to be great.

"I love as much as any man does the sweetness of Rousseau's soul. But it is not through men like him that the world is sustained. The world is a spectacle of moving beauty to one who is content merely to look upon it; but to one who wishes to participate in its affairs, it becomes a place of insipid beauty, a plaything for God's children only. Strive diligently and courageously to be benevolent. What would Luther or Vasa have amounted to if they had been like Rousseau? Rousseau, with the heart of a woman, did

nothing but complain all his life, and needed the support of all his friends to prevent him from succumbing to his infirmity. We can distinguish easily between the pretty fellows whose place is in a glass case, and those heroic souls created to avenge and maintain right." [16]

Still another element is to be observed in Thorild's soul-life; not so obvious, perhaps, nor so prominent as some others, but just as basic and genuine. He touches upon it in the self-characterization just given. Toward the conclusion he remarks that all his life he had worshipped "high-thinking." There were two tendencies active within him, two wills we might say, two lines of development, both of which struggled for realization and growth: the one, just mentioned, more active and sustained by a strong, unconquerable craving for expansion; and the other more passive, belonging to the retrospective life and serving merely as the reverse side of his desire for outward action. The latter wears a peculiar, dubious aspect, characteristic of all Spinoza-like souls, and is not immediately apparent. It is of an infinitely fine woof, spun of an impulse to philosophical examination and a conservative appreciation of beauty.

When we read Thorild, it is as if we were listening to the voices of two different persons. One is that of an aggressive, outwardly directed nature, a convincing preacher, full of a steadily progressive, steadily inspired life, a fighter and a dialectician who wields his weapons with incomparable precision and power. He wishes to recreate, to refreshen and to reform everything, "to uplift and ennoble." He desires to attain a trusted place by the side of the king, and only in exceptional cases does he seek shelter behind his "epicureanism," from the problems and duties of civil life. In some respects he held public functions and offices in high esteem, and muses with more or

[16] *Thomas Thorild's Letters*, p. 113ff.

less indifference upon the possibility of attaining a position of influence, feeling no insurmountable aversion to the use of somewhat questionable means in gaining it. But sometimes we seem to detect behind the aggressive controversialist and the excited haranguer, another man with more peaceful countenance and more moderate gestures. His is a calm and restful spirit, not always inflamed to action. He is a dreamer and mystic, who becomes attached to cults of romantic friendship and confesses unreservedly his kinship with nature, his burning longing to be fused and united with her. To the tender heart he interprets the beauty and melancholy of the quiet woods or life tripping merrily over field and meadow. To become perfect means to him to participate in the heightening consciousness of the meditative life. He is possessed of the acute desire for solitude so characteristic of all contemplative natures, and seeks steadily to deepen and broaden his soul. A solemn, profound calm descends occasionally upon the utterances and sentences of this second man, while his experiences possess always something of the sanctity and harmony of the religious life.

Thorild's development proceeds between these two poles. His life is a struggle between these two conflicting forces common in some respects to all men. But it widens at this point just through that intensity with which both forces successively prevailed in his life. The second element in his nature grew stronger especially toward the end of the 1780's and thus there gradually begins that spiritual shifting which was due, on the surface at least, to his sentence and exile and which was accelerated by these two events. I believe we can speak of an actual change in Thorild's development from this time on, and in that sense his exile and the experiences he underwent during its course, mark the turning point in his life, the stage where "recognition" begins. There was as yet, however, no radical change in

temperament and mood, no sudden reversal and no crisis. But between 1789-1795 occurred that intellectual taking count of stock and those tragic conflicts which result in a man's shedding most of his illusions. In the well-known letter of introduction to Herder he enumerates first the most significant moments in his life and then says about himself finally (in the third person): "He discovered at last that fancy, that Hecate of humanity, was and is the only hellish witch of the world, and knowing that Herculean strength could not prevail against her, he sought to overcome her by the use of Orphic Art." Here we stand on the path which leads to that ironical sophistry and resigned wisdom which mistakes and disappointments force upon even the strongest characters. It was by this path that he was to reach that sad, final knowledge of his potentialities and limitations which prompted him to write the unforgetable confession: "Our wishes do not form the theme of the world's melody. And therefore to attain harmony within oneself is the greatest rule of life." [17] Thus it was in his inner self rather than in his relations to the world outside that he came to realize at last that adjustment which he mentioned in one of his letters to Peter Tham as his real goal in life. In my opinion this self-reflection, this demand for unity, critical examination, clarity and harmony is not the least factor in making Thorild far more important than as a mere historical figure. From it issues his practical philosophy as well as his memorable metaphysics.

ANDERS KARITZ.

[17] *The Reform of Universities through the Institution of a Chair in Harmony or Conciliation*, p. 14.

FREEDOM VS. DETERMINISM IN RELATION TO THE DYNAMIC VS. THE STATIC

IT IS the aim of this paper to show that freedom and determinism are mutually exclusive but dependent members of a logical parallelism and that taken together they constitute a species of the more abstract parallelism of the dynamic versus the static.

We have seen in a previous paper[1] that pervading various disciplines of thought are to be found various parallelisms such as those of anatomy and physiology, matter and motion, act and content, the finite and the infinite, the absolute and the relative, appearance and reality. All of these are but different interpretations of the static and the dynamic in different fields of science, philosophy and logic. Freedom, we posit, is the functional, dynamic, or action-aspect of that situation whose structural, static or content-aspect is determinism.

Freedom holds quite as important a place in logical thinking as does determinism. Indeed, were it not for freedom, determinism would be incomprehensible and *vice versa*. Moreover it would seem, theoretically, that freedom makes determinism possible and likewise *vice versa*.

If we examine any cause, whatever it may be, we find it possible to make two important assertions about it. First, the cause possesses within itself no inherent power or force whereby it creates the effect. In other words it is not an agent, endowed with mystical energy; it actually does nothing, for it is only a step, a stage, a period in a sequence,

[1] "The Static and the Dynamic in the Logic of Science," *The Monist*, XXXIII, p. 556.

by definition finite, limited, bounded, arbitrary and absolute. After all, it is but a tool and a harmless, inactive, inert, dead, static, apparent thing. Causes and effects, then, are the modes by which continua are conceived to exist; they are the fashions by which we describe continua, chop them into sequences, units or successive parts, each unit being a state or a structure of these continua.

Secondly, we find that in reasoning from cause to effect we behave in quite the same fashion as in building a yard from inches. We ignore the inches, jump over the arbitrary boundaries between them and make of the longer distance a continuum of motion from the first to the thirty-sixth inch. So, in reasoning from cause to effect, both data must be forgotten or defied since they are arbitrary and limited segments of a continuum.

If causes and effects, then, are but practical and arbitrary means of analyzing continua and if they partake only of the nature of structures there is implied the continuum unanalyzed or unbroken into static sections, *i. e.*, the dynamic of which the causes and effects are the static. It follows, then, that the broken continuum pictured in terms of causes and effects, *i. e., determinism,* is in itself unbroken. Freedom represents this latter situation. Determinism, therefore, is a way of describing freedom; it is the mode by which freedom, as a situation, exists.

A further line of attack on the problem is found by the avenue of quantity versus quality. Quantities are but means of expressing qualities. They are the forms in which qualities exist. Qualities are but certain ways of defining existences, motions, actions—the dynamic. Quantities are but certain ways of defining states, units, parts, sections, and the like. A biological cell, for example, is in a last analysis nothing more nor less than a quantity. Parts or elements everywhere are theoretically reducible to quantities. Another example is the reduction of physical phe-

nomena to mathematical formulae. While space and time may not in themselves be numbers, the notion is apparently defensible that they are quantities, expressible in terms of numbers.

Abstracted from qualities, quantities are arbitrary and their arbitrariness makes it impossible to reason with them. They give us no information which as such warrants deductions. One gets from quantities no more than he puts in. As we have already seen,[2] in statistics one finds what he finds and that is all for figures represents facts as states or conditions and not processes of reasoning from or with these facts. Quantities everywhere are based, in the absence of qualities, solely upon the law of error. They are based on chance which is but another way of stating that they are arbitrary. Hence we can reason, predict, and derive one datum from another only in the realm of continua, actions or qualities, not with disjointed successions of compartmentalized units, amounts or quantities.

All of this means that when we are dealing with states or units we must remember that all we can say about them or rationally do with them is to admit that we find them as we find them. Now, causes and effects fall within this category of quantities; they are statistical tools; and just as we find it impossible to reason from quantity to quantity without admitting that the quantities are arbitrary and without resorting to a qualitative point of view, we find it impossible to reason from cause to effect. When we are apparently reasoning from cause to effect, we are predicting after the fashion of prediction from statistics. It is not a matter of deduction or induction but a matter of having a blind faith that an effect will follow from a cause since we have hitherto found it so. Rationally to think from cause to effect implies the arbitrariness of each, an un-

[2] "The Static and the Dynamic in the Logic of Science," *The Monist*, XXXIII, p. 556.

broken continuum, a quality of which the cause and effect are quantities.

Stated in other words there is nothing dynamic or qualitative about a cause any more than there is in a quantity. Determinism, then, means nothing more than our law of probable error—chance. We find sequences divided into observable parts. As far as our observation goes the sequences happen to be consistent. What is it then, that gives to the situation, determinism, its qualitative aspect? Freedom. Freedom supplies determinism with meaning just as reality supplies appearance with meaning or just as function supplies structure with meaning. Just as to reason from datum to datum requires a shift from a structural to a functional point of view, so in reasoning from cause to effect is required the shift from determinism to freedom.

Reasoning from cause to effect hinges upon the circumstance, then, that between the cause and effect there actually exists a continuum which is not destroyed by splitting it into parts for the parts are only apparent and not real. It now should be evident that we need a term to denote that continuum and to fit determinism as quality fits quantity and as the dynamic fits the static. We must provide determinism with a rational background; determinism must be supplemented by a coördinate meaning; logic demands that the reality "behind" the appearance of determinism be given a place in our thinking. We can make no exception of determinism unless it be our desire to relegate the notion of cause and effect to the scrap heap. It is evident that we reason from cause to effect by winking at the schism between them just as we imagine no boundaries to exist between the stages in a process of motion when we describe a continuum of motion. Freedom is this essential logical partner of determinism.

To review our point in a word, freedom is the reality by means of which we can regard determinism as an appearance. It is the theoretical situation upon which rests the practical notion of determinism. Freedom is the situation as it actually exists; determinism is the situation as it appears to exist; the former is end and the latter is means.

We now turn to meet a few possible objections to the abstract principles thus set forth. Is it not possible to conceive of a flexible unit, or relative cause and thus avoid the implied schism between a cause and its effect. Flexible units and relative causes are contradictions in terms. A unit is by definition fixed and absolute; it is never actually flexible or relative. Flexibility means change and a changing unit is the same sort of a creature as a live-dead person. To imagine a cause merging imperceptibly into an effect is likewise unthinkable for to do this makes of both data the same datum and destroys the unitary character of each. One might just as well insist that the quantity "2" is a relative matter and that it might be nearer three or nearer one, or that it is interchangeable with either three or with one.

It might be objected that causes and effects describe the static instead of the dynamic. Granted that a quantity implies a quality and that a state implies a state of something can we not have a quantity of a quantity or a state of a state? In other words is there not such a situation as a state of being? Quite obviously there is not. For example, let us assume that some water is only partly still. Stillness is a state and the "partly" is a state of the "stillness!" On closer scrutiny, however, it turns out that where the water is still it is absolutely still and where it is only partly still it is moving and therefore not still at all. There is no such situation, then as a state of being for a state quantifies an act, not another state. Thus determinism does not

quantify a state; it quantifies an activity. Causes and effects quantify continua; determinism quantifies freedom.

This means that anything is not free to be. Freedom implies and by definition means activity in the abstract. A state or anything structural is not free to be because it is not an activity. Being, as such, is inactive; its dynamic aspect is becoming. To become, then, is free and to be is determined.

There are many irrational statements made concerning our scientific god—cause and effect. One of these is the assertion that if we knew all causes we could predict all effects. This is idle talk. One does not know all causes without at the same time knowing all effects because any example of the former is thinkable only in relation to an already known effect. Further, knowing all causes would mean that there is nothing to predict from them. Thus to know all causes means to know no causes. This situation is but the logical parallel to the statement made a short time ago to the effect that we can derive nothing from quantities which we do not by definition put into them. Neither analysis nor synthesis is here possible.

Another statement often heard in connection with causation is equally as meaningless, namely, if a thing is caused it could not have happened in any other way. The notion that one is free to do as he chooses or that an event could not have occurred in any other way, given all of its causes, is superfluous talk from a logical standpoint. In the realm of cause and effect, choices are but after-thoughts; causes are not little tin gods nor are they little personalities possessing wills. The question "can or cannot" is a remnant of medievalism and a lingering of the idea that we are responsible for original sin. The problem "can or cannot" refers to abilities, potentialities not to causes and effects. It belongs neither to the problem of determinism nor to freedom.

Thirdly, the logical necessity which lingers as a halo about the notion of cause and effect is another much overworked notion. Necessity comes not from being caused but rests upon the common ground upon which the notion of cause and effect itself is found to rest, namely, in the so-called necessity of thinking about functions in terms of structures, of the infinite in terms of the finite and of the dynamic in terms of the static. The ways of action are by definition absolute and final; they correspond to mere observations; and if next time an event takes place differently than it did before its way has become another way, again absolute and final—but apparent and arbitrary. If it could have happened differently there is at once implied the antecedent cause and another observation and so on, *ad infinitum.*

One may very well contend that this is too abstract a notion of freedom. But after all this freedom is as definite as is all function, all motion or anything dynamic. We can comprehend the dynamic only in terms of the static for the latter is the way in which the former exists. We can comprehend the real only in terms of the apparent for the same reason. We comprehend the dynamic also in the sense that we *are* the dynamic. We feel it and live it and we act our freedom.

To summarize our discussion so far determinism describes freedom and freedom is the situation quantified by determinism. The latter is the dynamic; it is quality as offset against quantity; it is the real as offset against the apparent; it is end as offset against means; it is the relative as offset against the absolute; it is the infinite as opposed to the finite; it is a continuum as offset against a discontinuum. It is the "taking-place" of determinism. To express freedom necessitates determinism as to express a quality necessitates a quantity. Determinism is as baron without freedom, as structure is without function or as

matter is without motion or as the finite is without the infinite.

We have thus placed freedom in the category of functions. The terms of any of the static members of our pairs are interchangeable for they all have a common meaning; likewise are the terms of the dynamic members of our pairs. Action, or the dynamic *per se* is free; it is a continuum, infinite, relative and real. On the other hand, states, ways, fashions, parts, units, quantities, properties are absolute arbitrary, apparent; they are discontinua, all determined, *i. e.,* they follow the law of chance—cause and effect.

II

There are now a few approaches to the problem of freedom which are a little more specific. We must bear in mind that the freedom for which this paper argues is not freedom of choice for there is no choice, except in a practical sense, but a genuine freedom of action. Some of these more concrete arguments for this sort of freedom are as follows:

1. Nothing is determined by its own nature. A thing and its own nature are one. Determinism implies two terms and a relation between them. The two terms are the cause and effect; the relation is freedom. A thing and its own nature, however, provide us with only one term. There is no relation between any thing and its own nature except the relation of identity; therefore it cannot be conceived either as being free or determined.

2. Freedom is exclusive of causation in that it represents a different point of view from causation. We have already seen that by definition an act is exclusive of a state. Now it takes time for a cause to produce an effect. A

sequence or an extended continuum in time is implied. Actions proceed with or in time; they exist only as they are taking place and they are taking place only now or under a condition comparable to our "now" of time. Taking place with time, then, the time required for the operation of cause is lacking. In this concrete sense, as well, action is free. Only as we arbitrarily bind past, present and future together in a longitudinal section or series is determinism an applicable notion. Technically, then, an event has been caused but is now free, while taking place.

3. Action is free by virtue of its continuity. To break this continuity into parts means to destroy the dynamic. This is unthinkable. Freedom of action is as indestructible and as impenetrable as its analogue, the continuum.

4. Action is free from restraint in that a restrained act is not an act at all but a state; for when an act is partially hindered it is not the act itself which is restrained but it is its manner of taking place. To take place at all means to be that much independent of restraint. Motion must be motion not half motion and half stillness.

5. Neither freedom nor determinism involves the notion of intervention of free causes. In the first place there are no free causes and in the second place were intervention from without the realm of cause and effect thinkable we would find it thinkable only as we extended the realm and so on *ad infinitum*.

6. An act which is free in itself has no efficacy unless it functions as the cause of another act. Not only does this relegate all free acts to the category of caused acts as we shift our point of view from function to structure but it implies something more. An act which functions as a cause must have something in common with the act which it causes. This something-in-common is quality of some sort which we envisage in terms of a continuum. The acts must be alike in some way; a cause and its effect must have some-

thing in common. They must be reducible to common ultimate ingredients. In physics, these ingredients are construed as whirling particles; in psychology we construe them as kinaesthetic processes; in the realm of cause and effect we posit the unbroken continuum—freedom.

7. Let us imagine that we have the series of events A, B, C, D. We observe that A causes B. How possible? By means of a continuum involving a shift of view-point from structure to function. But a continuum as such admits of no A, B, C, D; there are no parts in a continuum or else it would not be a continuum. Paradoxical as it all may seem, two opposing points of view, two radically different concepts are necessary to complete the meaning. Determinism pictures the series of events, A, B, C, D, made possible by the imagined or implied continuum which by definition admits of no parts. Freedom pictures this continuum.

III

Turning now to a consideration of the human will we find that we can say of it what we said of action in general. The human will is action; it is to human behavior what motion is to physical data and what physiological concepts are to biological data. Any will act, conceived as a unit or as a state in a succession of acts is determined or caused; the fashion in which any will act takes place is likewise caused for its mode of taking place is a static concept. What we do or think, how we think and act, when we act, the direction of our movements, every feature or property of willing is determined. But will as action is free.

Human willing is free in an additional sense. Complexity of causation multiplies the chances for freedom. In human behavior is found a state of affairs which may be

likened to catalytic agency—the presence of memory or reflection. We repeat similar acts over and over again but as a result of this repetition the mode of the act is changed. B is A repeated but having performed A before the procedure in performing B is modified from the procedure in performing A. C is a further repetition; now both A and B have contributed to C; D is another repetition and is a product of A, B and C. Thus an act introduces into a causal sequence something which was not there before, namely a new set of relations between the repeated act and other acts. One act, repeated several times becomes easier than another while at the outset the other might have been easier. In such a complicated situation as is found in behavior this accumulative or additive nature of causes and effects gives to human conduct an element of auto-determinism after the fashion of a furnace and a thermostadt. Hence in a system which is from one point of view entirely mechanistic there is a legitimate place for effort and for trying. Effort is in itself an act, the causal functioning of which is accumulative. It contributes to the future behavior of the mechanism which is performing.

Now memory and reflection are by-products of our original acts. They are incipient repetitions of former acts and induce a parallel sequence of contributing causes which exaggerate the accumulative effect of habit. Here is a simple example: Today you choose Il Trovatore in preference to Aida; later, given the same alternatives, you choose the other, having heard Il Trovatore the time before. The old man tells the same story over and over again, having forgotten that he has told it before; while if he had remembered, he would have selected a different tale.

Memory and reflection operate in a singular fashion for while an act is taking place we tend to repeat it in incipient fashion; this is what is known as being conscious of the act. Not only do the changed relations introduced by re-

peating the original act appear at once in behavior as forceful causes which contribute to future conduct but consciousness of the act becomes an additional cause. Changed relations here appear in no other form than that of "meaning." In other words the comprehension of an act is that act repeated in incipient form and already delineated in its new set of relations to other acts. It is already operating as a new cause (new because of new relations) never before present.

Choice is merely the situation in which causes conflict. Antecedent causes of conduct have accumulated to the extent that they inhibit one another and make sudden adjustments impossible. Additional causes must supplement the old. The choice is but the readjustment process going on among the causes. Alternative courses of action are here made possible by a multiplicity of causes. But given a multiplicity of causes there is possible a multiplicity of effects until the multitude of causes is reduced to one.

From a quantitative standpoint, complexity of causation multiplies the chances of freedom. The chances for action are greater; the numbers of actions larger. Choice is only an incident to this fact.

A word, in closing, as to the ethical implications of the problem. We are prone to ask the question: Am I free? Is the individual free? But who are you and who is the individual are questions which must be answered before a reply can be given to such queries. "I" and the "individual" are static if regarded as entities and neither freedom nor determinism applies to them for each is but one term of a relation. Regarded as members of a sequence or as modes of function of something else whatever that may be, the "I" and the "individual" are determined. As activities they are free but from a practical standpoint, of course,

their freedom obtains only when their individuality is lost in a continuum or they are free when envisaged as compound action constantly taking place in a continuum of "nows." This latter is exactly what we are. As action, then, the individual is free.

Any act, then, is determined as long as it is regarded as a part of a complex of actions or as a stage in a series of actions; any act regarded as theoretically undivorceable from a continuum is free.

Responsibility and blame are legitimate only in a mechanistic system in which causes can be ascertained. The dilemma that we are not responsible unless free and are not responsible if we are free is due to the fact that we have not clearly understood the nature of consciousness and what the individual actually is. He is a complex mass of actions and reflection and memory are parts of these actions. These actions influence one another. Blame thus becomes useful as a feature of reflection and as a cause or deterrant of future acts. It is a mechanism whereby a meaning which failed to work once may be strengthened to work later. It is but another example of the many ways in which auto-determinism comes with increasing complexity of action. In the absence of such a situation as could or could not, the sense of blame operates as a contributing cause in a deterministic system. It is part of the mechanistic system itself.

It turns out, then, that the human will is both free and determined. It is determined, structurally and free functionally, as everything is. Freedom of the will is the will existing; determinism of the will is either the method or procedure whereby will acts take place or it is an act regarded statistically as a member of a sequence. Determinism of the will is the way in which freedom of the will exists. Freedom is that human action which is described or quantified in terms of cause and effect.

Freedom in the abstract is the "thing" of which determinism is a state or a property. Determinism is freedom's being; freedom is determinism's becoming. Determinism is appearance; freedom is reality.

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CRITICISMS AND DISCUSSIONS

MR. G. E. MOORE'S DISCUSSION OF SENSE DATA

WHATEVER one's point of view in philosophy, one can hardly fail to respect the spirit in which Mr. G. E. Moore discusses its problems. Rarely are to be found such high standards of earnestness, lucidity and scrupulousness as he brings to philosophical writing; and whether or not one agrees with the narrower program of investigation, to which such a method as he seems to propose might limit philosophy if adopted to the exclusion of other methods, one can not but acknowledge that his ideal is, in point of scientific rigor, unbiased candor and precision, worthy of philosophy in the truest sense. Yet admiration of Mr. Moore's powers of acute analysis need not act as a deterrent to others from engaging the same problems, even though they may despair ever of achieving his subtlety in drawing distinctions, for the very genuineness and impartial spirit of his inquiry rather provokes and invites further scrutiny of the problems to which he calls attention. It is, then, in the hope of exploring perhaps some further possibility that I shall undertake to consider Mr. Moore's discussion of sense data in his recent collection of essays,[1] and in particular the manner in which sense data may be related to physical objects.

Let us suppose, as Mr. Moore supposes, that I am looking at two coins, one a half-crown, the other a florin, both lying on the ground some distance away. As both are situated obliquely to my line of sight, the visual sense data (or "sensibles" as Mr. Moore calls them) which I "directly apprehend" in looking at them are elliptical rather than circular. In addition, the half-crown is farther away than the florin so that its sensible is visibly smaller than that of the florin. Now without bothering to define, if we could, what we mean here by physical objects, and without describing "sensibles" further than by saying that: they are all those entities, whether experienced or not, which are of the *same sort* as those that are experienced in

[1] Moore, G. E., *Philosophical Studies* (1922), Ch. V.

experiences of images, sensations proper, the sensory part of dreams, "after-images" and hallucinations, let us see if there are any propositions which can be asserted as true about these physical objects, and if so, in what sense they are true, and in what manner the objects are related to certain sensibles. To begin with, Mr. Moore assumes that he knows the following propositions to be true, since "no one," he says, "will deny that we can know such propositions to be true,"[2] although there are very different views as to their meaning: (a) that I am *really seeing two coins;* an assertion which at least amounts to the statement that the visual experiences constituting my direct apprehension of the two elliptical patches of color *are* "sensations proper" and not hallucinations or mere images; (b), (c), (d), (e) that the upper sides of the coins are *really* approximately circular, although the visual sensibles are merely elliptical; that the coins *have* another side, though I don't see it; that the upper side of the half-crown is really *larger* than that of the florin, although its visual sensible is *smaller* than that of the florin; lastly, that both coins continue to exist even when I turn away my head and shut my eyes. Obviously all these propositions, which we have here grouped together have to do in one way or another with my certainty that there is a distinction between sensibles and the physical objects themselves; while the last especially emphasizes my certainty that the objects continue to exist even when the experience of the sensations proper ceases. But let us stop to consider whether one is really justified in assuming that he knows the foregoing propositions to have absolute certitude.

First of all, am I right in assuming, as Mr. Moore assumes, that I know the proposition to be true that (a) *I am really seeing two coins* in the sense of experiencing sensations proper? Can I, in any given experience, accept as absolutely certain that the sensibles I "directly apprehend" are those of sensation rather than of hallucination or mere image? On the contrary, as it seems to us, the evidence clearly indicates the very opposite, *viz.*, that one has no right to assume that he knows any proposition to be true, in which statement is made distinguishing the sensibles of a present experience as of *one specific type* rather than another. All sorts of considerations bear this out. There is the evidence, for instance, of certain psychological experiments that, under proper circumstances, even trained observers are unable to distinguish in their experience between sensations and mere images. Thus it was found that if under elaborately

[2] *Ibid.*, p. 186.

controlled conditions, an observer was asked to fixate a certain point and to imagine a banana, while at the same time the outline of a banana faintly colored by a projection lantern was presented to him at that point on a dark screen, he almost invariably mistook the perceived outline of the banana for the mere image of his imagination.[3] Most of us can recall analogous examples from our own experience, as, for instance, where we have asked ourselves whether a barely discriminable pain was real or imaginary, or again (to borrow an illustration from Mr. Russell) when we have stood listening to a horse trot away along a hard road. For a time the listener's "certainty" that he still hears the hoofbeats is very strong; but there comes a moment when certainty merges into uncertainty, when he thinks perhaps it is only his imagination or his own heartbeats. Indeed, it seems probable that there are far more experiences than we suspect in which we are really incapable of distinguishing between sense data of different types; but, however that may be, the very fact that there are *some*, shows that we have no right to assume as unqualifiedly true a proposition which asserts that the sensibles occurring in a specific experience are exclusively of a particular sort. It may well be that what defines sensibles is fundamentally some *common* property shared among them; but whether, in experiencing a given sensible, that sensible is "directly apprehended" by me specifically as a *sensation proper* or as a *mere image* would seem to be a matter determined not by the experience of the sensible itself but through the formulation of some judgment or proposition *about* it, which attempts to express the status of the particular sensible with respect to physical objects, to which status extremely different interpretations may be given.

Indeed, it may be questioned on other grounds whether we are ever justified in affirming that we know a proposition to be true respecting particular sensibles of our experience, not merely because we are always liable to be mistaken as to the specific type of data in the experience, but also because it is a mistake to believe that the subjective "certainty" which we feel in our direct apprehension of sensibles can afford any ground for asserting the objective truth of the proposition in which the experience is described. The mere fact that mind seemingly acquiesces in the incursions of the data of sensory experience (neither volition nor intellect having the power to banish them) is something very different, of course, than if sensory experience carried in it some universal and necessary evidence

[3] Perky, C. W., *American Journal of Psychology*, Vol. 21, pp. 418-454.

of its veracity, which alone would entitle the mind to assume the truth of a proposition affirming the existence of certain particular sensibles in experience. The foregoing considerations apply equally well, moreover, when we pass to the group of propositions (b, c, d, and e) which assert in one form or another the distinction between sensibles and physical objects. For as these propositions have also to do with affirming certain sensibles to be of a specific type, we can not justifiably assume their truth, since any proposition which classifies the sensibles of a particular experience as of a certain kind is always open to falsity, and the "truth" which I ascribe to them is rather the expression of my subjective belief, arising from the unquestioning receptivity and submissiveness of the mind to sense data, itself a form of psychological response, and qualitatively quite different from the apprehension of valid relations between entities.

But even if, unlike Mr. Moore, we find ourselves unable to assume that we know any of the foregoing propositions to be true, we may none the less proceed to ask if there is any possible way in which the relation of sensibles to physical objects can be truly stated? Mr. Moore suggests four possible ways (which, however, finally reduce to two) in which sensibles can be related to physical objects, and these we may briefly consider. The first sense in which I might conceivably be able to affirm true propositions about physical objects would be *provided* they expressed the notion that *"if* certain conditions were fulfilled, I or some other person, *should* directly apprehend certain other sensibles."[4] Thus, on this interpretation, what is meant by saying that I really see two coins is some such thing as that *"if* I were to move my body in certain ways, I should directly apprehend certain *other* sensibles, this time tactual, which I should not apprehend as a consequence of these movements, if these present visual experiences of mine were mere hallucinations." If this view be accepted, the truth of propositions concerning physical objects is to be construed in terms of the hypothetical experiencing of certain hypothetical sensibles. Now although at the present time this form of explanation undoubtedly enjoys great vogue, the whole body of scientific laws being frequently so interpreted, there remains against it one serious objection: the expressions describing the coins and other physical objects as existing before I saw them can only be really true on this interpretation, if they are understood in an outrageously Pickwickian sense. In other words, the only possibly true construction of them will have to be one not only most uncommon

[4] Moore, *op. cit.*, p. 189.

but in which we are privileged to say one thing and mean another, and even to state under the form of fact something, for the present at least, quite contrary to fact, if indeed not outright contradictory. Thus all I can mean when I know that the coins existed before I saw them is that "*if* certain unrealized conditions had been realized, I should have had certain sensations that I have not had." [5] The difficulties here are readily apparent, since, on the one hand, the assertion that the coins exist really tells us nothing that would distinguish the objects themselves from the mere sensibles; while, on the other hand, it embodies a statement not only contrary to fact but resting upon unknown conditions, so that I am really declaring that "*if* certain unknown and unfulfilled conditions were fulfilled, and *if* I were then to experience sensibles which I do not, they would be sensibles of a certain sort,"—an assertion whose ***truth*** I have absolutely no grounds for affirming. For these reasons it would appear to us, as it also seemingly appears to Mr. Moore, that the interpretation of propositions about physical objects in terms of the hypothetical experience of sensibles would have to be rejected. This rejection would carry with it, moreover, dismissal of the second and third possibilities suggested by Mr. Moore, since they, too, are shown finally to fall back upon a hypothetical and Pickwickian interpretation.[6] The fourth and last possibility offered by Mr. Moore is one which he describes as "roughly identical with Locke's view." According to it, physical objects with certain qualities exist (in the natural sense) prior to my experience of them. Although not composed

[5] *Ibid.*, p. 191.

[6] One of these is the view that would interpret each particular physical object as being the "*cause*" of the experience of certain sensibles. But exception may obviously be taken to this, on the ground of its involving hopeless complexity, since in the example of the half-crown, for instance, the events which happen between the half-crown and my eyes, as well as events in my eyes and optic nerves, are just as much *causes* of my experience of the sensibles as is the coin itself. If an endeavor be made to meet this objection by saying that the half-crown has a particular *kind* of causal relation to my experience of certain sensibles, being, in fact, their "*source*" (and a source either "spiritual" or "unknown" in its nature), still nothing is really gained by this, since the only possible sense in which the physical object can here be said to have qualities is in the last analysis that of the Pickwickian interpretation. Another possible view would be frankly to describe the "source" of our experience of sensibles themselves; these latter existing even when not experienced. Nevertheless, under the proper conditions these unexperienced sensibles *would* be the source of our experiencing certain sensibles, etc., but since this seems all their meaning we here recognize only another variation of the Pickwickian theme (pp. 192-194).

of sensibles, physical objects do really *resemble* some sensibles in respect of the "primary qualities" which the latter have. The sensibles themselves, however (whether experienced or unexperienced) can never exist as *parts* of the objects or "anywhere in space" or "in the mind." Against this view, the most serious objection is that it does not make clear *how* we can ever come to know that sensibles have a "source" at all, or that this "source" may resemble sensibles as regards their primary qualities. Such knowledge, if we have it, would seem to have to be immediate. Yet can we know this kind of thing immediately? Our first argument certainly assumed that the only possible kind of immediate knowledge is that which we have in the direct apprehension of sensibles and in the perception of relations between directly apprehended sensibles. It follows from this that, if we believe we know facts other than these, and which can not have been learnt immediately, our belief must be a mere prejudice. Still, on the other hand, how can it be shown that our belief, that the only facts we can know immediately are sensibles and their relations, is not itself a mere prejudice? Certainly we have all of us, like Hume, a "strong propensity to believe" that physical objects exist in a simple and "natural" sense, and not merely in a Pickwickian one. And while this propensity to believe may be really only a prejudice, its strength seems so much greater than that of the prejudice that opposes it as to incline Mr. Moore apparently in its favor and in favor of the fourth view, which follows Locke's interpretation.

In conclusion, encouraged by the fact that Mr. Moore has sedulously left the question open, we may venture to suggest a further possibility. To us the first three views appear untenable because of the contradictions involved in any attempt to construe physical objects in terms of the hypothetical experience of sensibles under hypothetical suppositions and unexperienced conditions; while the fourth seems hardly more satisfactory, since it rests apparently on a simple "propensity to believe" in certain things on trust without being able to adduce rational justification of our belief; and although we may repose in such belief in certain moods of common sense, we seem liable to withdraw our assent in moments of critical reflection. Would there not be possible, however, a view of physical objects which should center in necessary and indubitable considerations? For there are certain incontrovertible elements common to all interpretations of physical objects, which, as it seems to us, might well be taken as constituting the sense in which the existence of physical

objects in any sense (and that we do in *some* sense is here assumed without discussion), we must at least affirm their subsistence as entities to which the "laws of thought" and the principle of uniformity[7] apply. In other words, physical objects can not be assumed at all without at least according them "being" in the minimal sense of assuming them to *be what they are* and that they behave uniformly. But if "existence" means fundamentally this, it may be said, it has no specific meaning, for the laws of thought apply to all entities in the universe, and hence existence as a predicate fails to denote one thing more than another; in this sense, sensibles, for instance, would exist exactly as precisely as much as do physical objects.

The whole question is so hedged with difficulties that we can only venture certain tentatives. It seems, however, that when we compare sensibles in their ordinary meaning with physical objects, they do not have identity in the same full sense as the latter. For although the laws of thought always apply to sensibles, sensibles, as compared with physical objects, are always *relatives;* they do not seem to have an identity of their own fundamental to their relations, but *what they are* is determined by *what they are related to.* With physical objects in general, however, this would not ultimately seem to be the case. We conceive their nature as in some sense being and remaining what it is fundamental to their connections, in spite of the fact that our particular views about physical objects are undoubtedly modified by taking them in these different connections. Whereas mere sensibles would seem to be defined relative to possible experience, to be capable of an endless variety of interpretations and without anything fixed and binding in their content, the physical world itself, on the other hand, would appear to be at bottom something necessary and determinate. What the mind ultimately seeks and finds in such a world is uniformities, necessary connections, in a word, an "*order* of nature." Sensibles, in so far as deductions and systematic meanings can be discovered through them, may be said to be reclaimed from their *prima facie* status as relatives whose opposite seems always possible to a place in an order that holds of all possible worlds. When I look at the two coins, for example, I apprehend certain sensibles which I recognize as relative to an indeterminate number of factors of experience, and as liable (through some

[7] If physical objects are subject to the laws of thought, they are certainly also subject to the principle of uniformity, since it would be nothing less than a denial of identity and the assertion that a thing *need not be what it is* to suppose that under the same conditions an object could behave in different ways.

slight alteration in these factors) at any moment to utter change. At the same time, I recognize that there are universal conditions governing these sensibles which determine them to be as they are and that any event that occurs under conditions with which its operation is connected universally. Insofar therefore as I refer these sensibles to determination through a totality of necessary conditions, I may be said to refer them to a physical order. Although I have no right to affirm that there actually are two physical objects of a certain kind in existence which resemble and correspond to these sensibles, I have at least the right to refer these sensibles with assurance to some general basis in a universal, orderly arrangement.

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NOTE ON TWO SCIENTIFIC ANNIVERSARIES

TWO men whose achievements hold the continuity of scientific attainment in Italy and have an integral place in the history of science in general are by the dates of their death and birth respectively worthy of special commemoration in the current year. Three hundred and fifty years ago Bartholomeo Eustachio died at Rome; just a hundred years later his semi-namesake, Eustachio Manfredi, was born at Bologna. A glance at their lives and works gives a bird's-eye estimate of the state of European culture as it emerged from the gloom of the Middle Ages into the sunlight of the Scientific Era.

Eustachio, that unfortunate man plagued by gout and poverty, whose service to a cardinal and professorship at the Studeo della Sapienza never advanced his fortunes, and whose intimate career is so little known that even the date of his birth is uncertain, was nevertheless one of the great founders of modern anatomy. With Vesalius (against whose attacks he defended his master Galen, then with characteristic unpracticalness changed his mind when he had succeeded in making Vesalius his permanent enemy), and with Fallo-

pius. Eustachio makes up the Anatomical Trinity from whom the modern science had its birth. No da Vinci, his plates nevertheless had scientific accuracy and some artistic merit. Son of a physician, of a family of physicians, his path was foredained. The kidneys, the teeth, the muscles of the head and neck, a dozen other portions of the human body received his scrutiny; but his greatest work was never finished—poor Eustachio, that might have been expected!—and the plates he had prepared for it were not discovered and published until 1714. Long before then he had become to the layman merely a name—godfather of the Eustachian tube between the throat and the ear—and to the anatomist little more than another name—discoverer of the Eustachian valve of the foetal heart.

Eustachio died in August, 1574; the scientific world rolled on, rather faster in Italy at the time than elsewhere in Europe. By September, 1674, its chief interest was not anatomy, but astronomy and physics. The epoch-working book of Copernicus had made its appearance, with its author's death, thirty-one years before Eustachio resigned his unhappy life; Galileo had been dead thirty-two years, and was shouting "Eppur si muove" down the corridors of time, when Manfredi was born into a new and changing world.

The parents of young Eustachio Manfredi may have betrayed a latent scientific interest by the name they gave their son; but he was commanded to the study of law. He obeyed—and entertained himself with mathematical problems while his fellow-students reflected on rights and damages. At last his bent became too powerful; he was permitted a transfer to the mathematical department. The boy gathered the like-minded among his fellows into a group, a sort of little society which met and discussed scientific questions. From it grew the Institute of Bologna, destined to be the breeding-place of many learned men. More—he built himself an observatory and a telescope, and taught himself astronomy.

At twenty-four he was professor of mathematics in the university where he had studied; six years later he became director of the College of Montalto. He died in the city of his birth, February 15, 1739, leaving as his chief contribution the famous *Ephemerides motuum celestium,* which he carried on from 1715 to 1725, and which his successors calculated for many years afterward. Manfredi's own conclusions and speculations were expressed in the introduction to that work—*De novissimis circa siderum fixorum errores observationibus epistola.* Rather timid conclusions they were—their author did not court martyrdom, and he wanted to remain in his

professorial chair. So he flirted coyly with the idea of the motion of the earth round the sun—left it to be inferred—and refrained from declaring himself explicitly.

Therefore, he died at a fairly ripe old age and honorably reputed. The great authority on hydrostatics of his time, he was a commendable worker also in geometry, astronomy and gnomonics (the superceded science of the sundial and the calculations made by it). One wonders if his present fame would not be more, had he announced himself an outright Copernican.

Two hundred and fifty years from now, analogous inquiry will doubtless be made of some learned professor of our own day, confronted by a similar invitation to a contemporary martyrdom. It is safe now to say that the earth revolves around the sun—but there will always be plenty of unsafe things to tempt and alarm the professors!

MAYNARD SHIPLEY.

BELL AND WINCHELL: REFLECTIONS AFTER A CENTURY

IT is curious to observe the effects of mere *period* in history on the thought and work of men who, had they chanced to be contemporaries, might have been much more akin in point of view. A difference of fifty years in birth sometimes makes a difference of five hundred in mental approach to similar problems.

The accident of a centenary brings this out strikingly in the case of a Scotch anatomist and an American geologist—Sir Charles Bell and Alexander Winchell. Both were close students—Bell the superior in original research—both born teachers, both influential chiefly in personal contact with those they taught. But—Bell was born in 1774 and Winchell in 1824. Half a century created an almost unbridgeable gulf between their attitudes.

Because Bell annotated and illustrated an edition of Paley's *Natural Theology*—that good old stand-by of the special creationists; because he believed firmly in the doctrine of design in the universe,

and wrote and spoke all his life in its favor, it must not be forgotten that he conferred inestimable benefits on science. The consequences risen from it by men working in opposite directions to his own, would have shocked and disturbed Charles Bell mightily; fortunately for his peace of mind, he was dead by 1842, seventeen years before *The Origin of Species* startled the world, and his work remains, however he would doubtless prefer to see it destroyed in this godless and designless generation.

"You do not need any lecture today; you have seen Charles Bell," said a French professor to his pupils when the great anatomist had honored the class with a brief visit. Bell might have deprecated the flattery, but he would have agreed as to the uselessness of much formal education. "Nonsense!" he wrote, when a tablet announced his education in the Edinburgh High School. "I received no education but from my mother!" The doughty Scotsman, son of an anatomist locally famous in his time, fell upon the study of the human body as if by predestination. When Fellows of the Royal College of Surgeons at Edinburgh were no longer allowed to act as surgeons at the Royal Infirmary, Bell offered a hundred pounds and the museum he had collected for permission to "stand by bodies when dissected and make notes and drawings." Refused, he removed himself and his museum to London, where both had a great success. It was only toward the end of his life that he consented to return to his native Edinburgh, for "London," said he, "is a place to live in, not to die in."

His *Anatomy of Expression*, published soon after his removal to London in 1804, with its plates of his own making, illustrating the muscular movements accompanying the various emotions, has been a source book ever since for anatomists, surgeons and artists. But his great work was the discovery that the functions of the nerves corresponded with the different parts of the brain, and the division of the nerves, according to function, into sensory, motor and sensory-motor. Bell was first to show that the anterior roots of the spinal nerves are motor, the posterior sensory, and that no motor nerves pass through a ganglion. For such a discovery much may be forgiven him—even the sponsoring of a new edition of Paley. His theory of creative design was based on science as sound as his age afforded—he did not subscribe to the worst extravagances of his school, like the gentleman who pointed out the greatness of God's wisdom: He had put holes in the cat's fur just where the eyes were!

When Sir Charles Bell had reached his half-century mark, Alex-

ander Winchell was born in New York State, on the last day of 1824. His career until his death in 1891 was a succession of professorships—yet nothing farther from the dry-as-dust professor could be imagined. Geology was his special subject, but he taught botany and zoology as well—even physics and civil engineering as his first terms at Michigan—and he did yeoman service in popularizing, by books and lectures, the evolutionary theory, one of whose earliest spokesmen he was. His *Reconciliation of Science and Religion,* his *Preadamites,* his *Sparks from a Geologist's Hammer,* his *World-Life,* fired the imagination and determined the bent of more than one scientist whose youth lay in the years immediately following the Civil War.

Meanwhile Winchell proceeded on his honored way—state geologist of Michigan, director of the Michigan Geological Survey, chancellor of Syracuse University, professor at Kentucky University, Syracuse, Vanderbilt, twice at Michigan, where he died. His brother Newton curiously duplicated his career in Minnesota, even to being state geologist and professor of geology at the state university.

But Winchell, too, had his set-to with the obstinately surviving spirit that had confused Bell's scientific viewpoint. For three years he served a double professorship, traveling from Syracuse to Vanderbilt, in Tennessee. Then the good churchmen who were Vanderbilt's trustees grew scandalized at the spectacle of an evolutionist wolf let loose among their lambs. Winchell was unceremoniously dropped from the faculty. He smiled, and went on lecturing and writing to popularize the doctrine which had caused his disaster.

As obscurantism, fed by post-war mysticism, spreads through our universities, especially in the south, where so much of Winchell's teaching was done; as the reactionary point of view of Bell, untouched by his scientific zeal, rises anew to battle with the forces of clear-sighted secular science, one longs for a new Alexander Winchell to lead the age-long battle. After all, a century is not a very long time for a world to move forward in!

MAYNARD SHIPLEY.

BOOK REVIEWS AND NOTES

SUBSTANCE AND FUNCTION AND EINSTEIN'S THEORY OF RELATIVITY. By *Ernst Cassirer.* (Pp. i-xii, 3-465). Translated from the German by W. T. Swabey and Marie C. Swabey. Open Court Publishing Co., 1923.

The development of the concept is adequate, according to the author of *Substance and Function,* to explain the totality of the metaphysical implications of science; epistemology, the nature of reality and being. To facilitate this aim, *Substance and Function* is divided into three parts: Part I, The Concept of Thing and the Concept of Relation; Part II, The System of Relational Concepts and the Problem of Reality; Supplement, Einstein's Theory of Relativity Considered from the Epistemological Standpoint.

Doctor Cassirer's starting point is a disparagement of the naive view of the world. In common with current idealists, he pronounces naive realism to be a wholly inadequate norm of expressing the summation of all possible connections in the causal chain. Thus he would discount the tendency of the layman and scientist alike to assume as a working hypothesis a one-to-one correspondence between his ideas of reality and the reality itself.

Conceptual synthesis of the entire field of mathematical and "natural" reality is the writer's controlling purpose. The main tenor of *Substance and Function* is therefore mathematical, for this method is of course the mind's only expedient of unifying static concepts dissociated from the current flow of reality. Our ideas of external reality are hence not direct copies of sense experience but are partly, if not wholly, contributions of our own mental makeup.

The unique feature of Doctor Cassirer's treatment is his attempted projection of the world of the abstract on outer reality. Series formulae are applied to such simple and fundamental laws in chemistry as the laws of definite and multiple proportions. In the introductory chapter, some attempt is made to determine how far into the range of natural reality mathematical formulation is legitimately applicable, and due recognition is accorded its present insufficiency. This apparent dissociation, or cleavage, between mathematics and blunt reality is to be attributed, according to the author, to the absence of a complete conceptual synthesis. This conceptual unity implies that knowledge is a closed system, a conceptual pyramid.

In passing, it is instructive to note that the experimental part of modern science has a unique verification and reality independent of any of our conceptual judgments regarding it. Present tendencies of theoretical physics to thrust the language of mathematics on physical phenomena have not seemed to eventuate in any great discoveries. An extremely profitable research would consist in an exhaustive survey of experimental physics with a view of estimating

approximately, at least, how applicable and genuinely stimulating the mathematical method has been in unraveling nature's laws. It would appear, in spite of countless attempts of philosophers and physicists alike to project the eccentricities of mind on nature, that empirical reality is of a totally different order, and is not amenable to such complex treatment.

The author has considered quite fully in their historical bearings such problems as (1) the generic concept, (2) the concept of number, (3) the concept of space and geometry, (4) the concepts of natural science from the point of view of mechanism and energism, (5) the concept of reality from the point of view of neo-realism and critical realism, and (6) the subjectivity and objectivity of the relational concepts. The supplement is a Résumé of the epistemological concerns of relativity and the applications of these ideas to philosophical concepts of truth. *Substance and Function* is on the whole an admirably complete survey of the purposes and tenets of the scientific philosophy, coördinated by conceptual judgments.

Granting the validity of the entire treatment, it would appear that the omniscience of mind in nature is extraordinarily absolute, since mathematics has gone ever so much farther than empirical science. Differential equations of low order and simplicity seem adequate to express the most complex phenomena of physics and chemistry, such as bi- and tri-molecular reactions, and the law of radioactive decay. The great bulk of the phenomena of Magnetism Light, and Electricity can be expressed by relatively simple formulae. No recourse need be made to such complex mathematical constructions as Group Theory, Riemann Geometry, Higher Algebra, and Hermitrian Matrices in General Analysis. Any attempt to apply these beautiful and infallible constructions of the mind to reality seems especially sterile, and leads to no experimental consequences of value.

From the point of view of possible experimental consequences, Doctor Cassirer's attempt to apply these infallible concepts of pure mathematics to the physical world seems fruitless. Insofar as mathematical philosophy fails to predict the course of an empirically determined chain of events, or to lead to workable results, its treatment is of little use. It is of course possible that the function of higher mathematics is to synthesize a series of empirical events and facts after they have been discovered, rather than to aid in the prediction of their discovery. Mathematical philosophy is in this sense analogous to the ether of space; it is a cohesive substratum or continuum which renders the separate discontinuous atoms of fact mutually compatible and interpenetrable.

Scarcely less fundamental than Doctor Cassirer's illuminating and helpful fruition of conceptual synthesis would be an attempt on his part to determine how far into the range of physical reality, his complete conceptual synthesis is quantitatively applicable. This is not his problem, it must be conceded, but it is manifest that such a research would do much to clear up the present solepsistic dilemmas of higher mathematics.

Substance and Function is unusually complete in what it purports to be: A résumé and coördination of scientific philosophies. Although Einstein's theory is treated at great length, scant mention is alloted Planck's ideas and the development of the quantum theory. Here, indeed, is a field for philosophical speculation which is far more promising and valuable than the construction of universal space and universal time. The problem of quantum theory in its wide

and diverse application to the nature of the interaction of matter and light, and the reciprocal transformations of matter and energy is a problem of far greater value than theories of four and *n*-dimensional manifolds. It is conceivable that a unificaton of these two theories will eventuate in their mutual development. In point of fact, certain evidence in the field of atomic structure has already given confirmation to the restricted theory of relativity. True enough, Doctor Cassirer has alloted Planck and the quantum theory a few incidental remarks, but nothing at all commensurate with the importance of this field of knowledge. With this possible exception *Substance and Function* gives a good summary of the philosophical developments in its chosen field.

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